Northern Virginia Community College Third Annual NOVA Mathematic Competition April 3, 2015 Competition Questions Round 1 Question 1 to 10

Team Number: _

20 people in Carl's math test take a probability exam, not including Carl, since he is sick at home. Their average score is 16. After Carl comes back and takes the exam, the average score in the test jumps to 19. What did Carl get on the exam?

The solution =

Official Use Only : Is answer is correct?

No

Team Number: _____

Which of the following five numbers is the mean of the other four numbers?

37, 49, 51, 53, 75

The solution =

Official Use Only : Is answer is correct?

No

Team Number:

A car lost 1/3 of its original value the first year of its life, it lost 1/4 of its original value the next year of its life, and it lost 1/5 of its original value the next year. After this time, it is worth \$4,043. What was the car's original dollar value?

The solution =

Official Use Only : Is answer is correct?

No

Team Number: _____

The base 2 integer 100101011 is converted to base 10. What will the integer be?

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Team Number: _____

The expression
$$2 + \frac{2}{2 + \frac{2}{2 + \frac{2}{2 + \frac{2}{3}}}}$$
 can be simplified to $\frac{a}{b}$

where a and b are relatively prime. What is a + b?

Question 5

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Team Number: _____

The sum of the slopes of the perpendicular line with equations y = ax and y = bx equals to $\frac{9}{20}$. Find |b - a|. (in simplified fraction)

The solution =

Official Use Only : Is answer is correct?

Yes

No

Team Number: _____

Find the coefficient of the *abc* term in the expansion of $(a + b + c)^3$.

The solution =		
Official Use Only : Is answer is correct?	Yes	No

Team Number: _____

From a standard 52-card deck, how many way to choose a set of 5 cards that has at least one face card in it?

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Team Number:

Suppose there is an island containing ONLY two types of people: knights who always tell the truth and knaves who always lie. You visit the island and are approached by two natives who say the following:

- A says: B is a knight
- B says: A and I are of opposite type.

What are A and B?

The solution = Official Use Only : Is answer is correct? Yes No

Question 10Team Number:What is the last digit of the sum 1! + 2! + 3! + ... + 2014! + 2015!?

Official Use Only : Is answer is correct?

No

Yes

The solution =

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Competition Questions Round 2 Question 11 to 20

Team Number: _

At 2:00 PM in one afternoon, a light pole casts a shadow on a man who stood 7 ft 5 in away from the pole. If the man is 6 ft tall and has a shadow length 4 ft 9 in, how tall is the light pole? (unit in feet and round the nearest tenth)

The solution =

Official Use Only : Is answer is correct? Yes No

Team Number: _____

The graph of xy - 6x + 4y = 36 is symmetric with respect to (p, q). Find the product of p and q.

The solution = Official Use Only : Is answer is correct? Yes No

Team Number: _____

If $\sin(x) + \cos(x) = \frac{1}{2}$, what is the value of $\sin^3(x) + \cos^3(x)$?

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Team Number: _____

The equation of $a^3 + b^3 + c^2 = 2015$ has exactly one solution in positive integers for which a > b. Find a + b + c for this solution.

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Team Number: _____

Omitted from the Competition.

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Team Number: _____

For the function f(x), f(1) = 3. Also, $f(x) + f(y) = f(\frac{x+y}{2})f(\frac{x-y}{2})$ for all real number x and y. What is f(6)?

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Team Number: _____

Is 3599 a prime number? If not, what are its factors?

The solution =

Official Use Only : Is answer is correct? Yes No

Team Number: _____

After surveying 150 people at the Cooking Convention, we obtain the following results on appliance brands as :

- 58 like Samsung
- 63 like GE
- 58 like Kenmore
- 19 like Samsung and GE
- 17 like Samsung and Kenmore
- 4 Like Kenmore and GE
- and 1 likes all three

How many people did not like Samsung, Kenmore nor GE?

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Team Number: _____

How many irrational solutions does the equation $18x^4 - 11x^2 = -1$ have?

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Team Number: _____

No

A unit regular hexagon ABCDEF is drawn on the plane. What is the area of ΔACE ? Express your answer as a common fraction in simplest radical form.

The solution =

Official Use Only : Is answer is correct? Yes

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Competition Questions Tie Breaker

Question Tie-Breaker 1

Team Number: _

Let A be a unit square. What is the largest area of a triangle whose vertices lie on the perimeter of A?

The solution =

Official Use Only : Is answer is correct?

No

Question Tie-Breaker 2

Team Number:

A parabola has a vertex at (0,8) and the x- intercept of 2. Find an equation of the parabola in a form of $y = Ax^2 + Bx + C$.

The solution =

Official Use Only : Is answer is correct? Yes

No

Question Tie-Breaker 3

Team Number: _____

If three digits distinct counting numbers chosen from the set $\{1,2,3,4,5\}$, find the probability that the product of the digits is odd. (in simplified fraction)

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No