Northern Virginia Community College Third Annual NOVA Mathematic Competition April 3, 2015
Competition Questions
Round 1
Question 1 to 10

## Question 1

## Team Number:

$\qquad$
20 people in Carl's math test take a probability exam, not including Carl, since he is sick at home. Their average score is 16. After Carl comes back and takes the exam, the average score in the test jumps to 19. What did Carl get on the exam?

The solution $=$

## Question 2

Team Number:
Which of the following five numbers is the mean of the other four numbers?
$37,49,51,53,75$

The solution $=$

Official Use Only : Is answer is correct?
Yes
No

## Question 3

## Team Number:

$\qquad$
A car lost $1 / 3$ of its original value the first year of its life, it lost $1 / 4$ of its original value the next year of its life, and it lost $1 / 5$ of its original value the next year. After this time, it is worth $\$ 4,043$. What was the car's original dollar value?

| The solution $=$ |  |  |
| :--- | :--- | :--- |
|  |  |  |
| Official Use Only : Is answer is correct? | Yes | No |

## Question 4

Team Number:
The base 2 integer 100101011 is converted to base 10 . What will the integer be?

[^0]
## Question 5

Team Number: $\qquad$
The expression $2+\frac{2}{2+\frac{2}{2+\frac{2}{2+\frac{2}{3}}}}$ can be simplified to $\frac{a}{b}$
where $a$ and $b$ are relatively prime. What is $a+b$ ?

| The solution $=$ |  |  |
| :--- | :--- | :--- |
|  |  |  |
| Official Use Only : Is answer is correct? | Yes | No |

$\qquad$
The sum of the slopes of the perpendicular line with equations $y=a x$ and $y=b x$ equals to $\frac{9}{20}$. Find $|b-a|$. (in simplified fraction)

## Question 7

Team Number:
Find the coefficient of the $a b c$ term in the expansion of $(a+b+c)^{3}$.

The solution $=$

Official Use Only : Is answer is correct?
Yes
No
$\qquad$
From a standard 52 -card deck, how many way to choose a set of 5 cards that has at least one face card in it?

The solution $=$

Official Use Only : Is answer is correct?
Yes
No

## Question 9

Team Number:
Suppose there is an island containing ONLY two types of people: knights who always tell the truth and knaves who always lie. You visit the island and are approached by two natives who say the following:

- A says: B is a knight
- B says: A and I are of opposite type.

What are A and B ?

The solution $=$

Official Use Only : Is answer is correct?
Yes
No

Question 10
Team Number:
What is the last digit of the sum $1!+2!+3!+\ldots+2014!+2015!?$

The solution $=$

# Northern Virginia Community College Third Annual NOVA Mathematic Competition 

Competition Questions<br>Round 2<br>Question 11 to 20

## Question 11

Team Number: $\qquad$
At 2:00 PM in one afternoon, a light pole casts a shadow on a man who stood 7 ft 5 in away from the pole. If the man is 6 ft tall and has a shadow length 4 ft 9 in, how tall is the light pole? (unit in feet and round the nearest tenth)

| The solution $=$ |  |  |
| :--- | :--- | :--- |
|  |  |  |
| Official Use Only : Is answer is correct? | Yes | No |

## Question 12

Team Number: $\qquad$
The graph of $x y-6 x+4 y=36$ is symmetric with respect to $(p, q)$. Find the product of $p$ and $q$.

The solution $=$

## Question 13

Team Number: $\qquad$
If $\sin (x)+\cos (x)=\frac{1}{2}$, what is the value of $\sin ^{3}(x)+\cos ^{3}(x)$ ?

The solution $=$

## Question 14

## Team Number:

$\qquad$
The equation of $a^{3}+b^{3}+c^{2}=2015$ has exactly one solution in positive integers for which $a>b$. Find $a+b+c$ for this solution.

[^1]
## Question 15

Team Number:
Omitted from the Competition.
$\square$
The solution $=$
$\qquad$
For the function $f(x), f(1)=3$. Also, $f(x)+f(y)=f\left(\frac{x+y}{2}\right) f\left(\frac{x-y}{2}\right)$ for all real number $x$ and $y$. What is $\mathrm{f}(6)$ ?

The solution $=$

Team Number:
Is 3599 a prime number? If not, what are its factors?

The solution $=$

## Question 18

## Team Number:

$\qquad$
After surveying 150 people at the Cooking Convention, we obtain the following results on appliance brands as :

- 58 like Samsung
- 63 like GE
- 58 like Kenmore
- 19 like Samsung and GE
- 17 like Samsung and Kenmore
- 4 Like Kenmore and GE
- and 1 likes all three

How many people did not like Samsung, Kenmore nor GE?

The solution $=$

Official Use Only : Is answer is correct?
Yes
No

## Question 19

## Team Number:

How many irrational solutions does the equation $18 x^{4}-11 x^{2}=-1$ have?

[^2]
## Question 20

## Team Number:

$\qquad$
A unit regular hexagon ABCDEF is drawn on the plane. What is the area of $\triangle A C E$ ? Express your answer as a common fraction in simplest radical form.

| The solution $=$ |  |  |
| :--- | :--- | :--- |
|  |  |  |
| Official Use Only : Is answer is correct? | Yes | No |

# Northern Virginia Community College Third Annual NOVA Mathematic Competition 

## Competition Questions <br> Tie Breaker

## Question Tie-Breaker 1

Team Number:
Let A be a unit square. What is the largest area of a triangle whose vertices lie on the perimeter of A ?

The solution $=$

Yes
No

## Question Tie-Breaker 2 <br> Team Number:

A parabola has a vertex at $(0,8)$ and the $x$ - intercept of 2 . Find an equation of the parabola in a form of $y=A x^{2}+B x+C$.

The solution $=$

## Question Tie-Breaker 3

Team Number: $\qquad$
If three digits distinct counting numbers chosen from the set $\{1,2,3,4,5\}$, find the probability that the product of the digits is odd. (in simplified fraction)

The solution $=$


[^0]:    The solution $=$

    Official Use Only : Is answer is correct?

[^1]:    The solution $=$

    Official Use Only : Is answer is correct?
    Yes
    No

[^2]:    The solution $=$

    Official Use Only : Is answer is correct?
    Yes
    No

