Northern Virginia Community College Fourth Annual NOVA Mathematic Competition April 8, 2016 Competition Questions Round 1 Question 1 to 10

Team Number: _____

A natural number is said to be perfect if it is equal to the sum of its proper divisors. Example 6 has divisors, are 1, 2, 3, and 1+2+3=6, so 6 is a perfect number. What is the next perfect natural number?

Г	The solution $=$		
	official Use Only · Is answer is correct?	Ves	No

Team Number: _____

Find a number 373982 base 10 convert to a number base 16 $\,$

. The solution = Official Use Only : Is answer is correct? Yes No

Team Number: _____

My son's piggy bank has 42 coins worth exactly \$1.00. If it has at least one quarter, dime, nickel, and penny, find the total number of dimes and nickels.

The solution =

Team Number: _

Tom spends \$25 on 3 CD's, each costing a whole number of dollars. The first CD costs more than twice the second, but less than three times the third; the second CD cost more than the third. Find the cost of the third CD.

The solution =

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Team Number: _____

Find the two complex roots of $x^3 + 1 = 0$.

The solution =		
Official Use Only : Is answer is correct?	Yes	No

Team Number: _____

The first two terms of are 10 and 20. The rest of the terms are the average of the preceding terms. Find the 2016^{th} term

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Question 7Team Number:Solve $\sqrt{x+7} = 2 + \sqrt{x}$ (must be in simplified fraction)

. The solution = Official Use Only : Is answer is correct? Yes No

Team Number: _____

Given: Julia's age is a 2 digit number. The remainder is 1 when her age is divided by 2, 3, 4, 6 or 8. Julia is 5 times Bart's age. Find Bart's age

The solution =

Team Number: _____

Find the last two digits of 2^{1000}

The solution = Official Use Only : Is answer is correct? Yes No

Question 10	
Given $x + \frac{1}{x} = 3$, find the $x^4 +$	$-\frac{1}{x^4}$

Team Number: _____

The solution = Official Use Only : Is answer is correct? Yes No

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Competition Questions Round 2 Question 11 to 20

Team Number: _____

A store sells 10 apples for \$6 or 20 apples for \$10. How much would you save by buying 25 apple at the 20 apple rate instead of 10 apple rate?

The solution =

Team Number: _

In right triangle ABC (right angle at C), points D and E lie on hypothenuse so that AD = DE = BE. If $CD = 10\sin(x)$ and $CE = 10\cos(x)$ for some value x, find AB

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Team Number: _

In a hand of 5 card poker, 5 cards of consecutive denominations, all in the same suit (excluding Royal Flush) are called Straight Flush. How many possible way to have a Straight Flush from a standard deck of 52 cards?

The solution =

Team Number: _____

Convert Roman number to our Natural number: MMDCXIV

The solution =

Official Use Only : Is answer is correct?

No

Yes

Team Number: _____

Find all x in degree such that $\cos^2(x) + \cos(x) = \sin^2(x)$ on the domain $0 \le x \le 360^o$

The solution —		
Official Use Only : Is answer is correct?	Yes	No

Team Number: _____

Counting possibilities on a combination lock. How many different three number combination are possible in a combination lock having 62 numbers on its dailed?

The solution =

Team Number: _

One way of find the next number of the sequence, mathematician uses a method called successive difference. Use this method to find the next number of the following sequence.

6,20, 50, 102, 182, 296, _____

The solution =

Official Use Only : Is answer is correct?

No

Yes

Team Number: _____

Find a domain of $f(x) = \ln\left(\frac{x^2 - 1}{x}\right)$ using interval notation.

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Question 19 Team Number: _____ Let r_1 be the remainder of $x^6 - x^4 - 3x^3 - 2x + 5$ divides by x - 1. Let r_2 be the remainder of $x^6 - x^4 - 3x^3 - 2x + 5$ divides by x + 2Find the $3r_1 + r_2$

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Team Number: _

Circle S has equation $x^2 + y^2 = 16$. If P(1,0), Q(-1,0), and \overline{R} is a point on a circle S. What is the largest possible value of PR + QR? (must be exact value)

The solution =

Official Use Only : Is answer is correct? Yes

No

Northern Virginia Community College Fourth Annual NOVA Mathematic Competition

Competition Questions Tie Breaker

Question Tie-Breaker 1

Team Number: _____

How many non repeating four-digit numbers have the sum of their digits equal to 30?

The solution = Official Use Only : Is answer is correct? Yes No

Question Tie-Breaker 2

Team Number: _____

Let x be a rational number. If x = 0.1563156315631563...What is the simplified fraction of this repeating decimal number?

The solution $=$		
Official Use Only : Is answer is correct?	Yes	No

Question Tie-Breaker 3

Team Number:

The students in Mrs. N class can be seated in rows of 4 or 5 each time with exactly the same number of seats in each row, but when seated in rows of 6, one row has exactly 2 fewer students than all other rows. If 4 new students join the class, in how many equal rows could her students now be seated?

The solution =

Solutions to Round1

- 1. Answer : $\mathbf{28}$
- 2. Answer : 5B4DE
- 3. Answer : **6**
- 4. Answer : **\$ 5**

5. Answer :
$$\frac{1+\sqrt{3}i}{2}$$
 and $\frac{1-\sqrt{3}i}{2}$

- 6. Answer : **15**
- 7. Answer : $\frac{9}{16}$
- 8. Answer : $\mathbf{5}$
- 9. Answer : **76**
- 10. Answer : 47

Solutions to Round 2

- 11. Answer : \$ 2.50
- 12. Answer : $6\sqrt{5}$ OR $\sqrt{180}$
- 13. Answer : **36**
- 14. Answer : **2614**
- 15. Answer : $60^{\circ}, 180^{\circ}, 300^{\circ}$
- 16. Answer : **226,920**
- 17. Answer : $\mathbf{450}$
- 18. Answer : $(-\infty,-1)\cup(-1,0)\cup(0,1)\cup(1,\infty)$
- 19. Answer : **81**
- 20. Answer : $2\sqrt{17}$ OR $\sqrt{68}$

Solutions to Tie Breaker

Answer : $\mathbf{24}$

 $\begin{array}{l} \text{Answer}: \ \frac{521}{3333}\\ \text{Answer}: \ 11 \end{array}$