Practice Test for Units 1 - 5

The problems here are similar to those on the online practice test.

1. Order from least to greatest
   30/7, 4 4/5, 21/5, 3 6/7

   The challenge in comparing these numbers is that they are in different formats. So put them into the same format. The first step in that direction is to convert the mixed numbers to improper fractions or vice versa. Let's make them all into mixed numbers. See the explanation for question 2 for how to make the conversion.

   Now this list looks like this:
   4 2/7, 4 4/5, 4 1/5, 3 6/7

   That clears up some of it. Clearly, 3 6/7 is the lowest number and it belongs first in the list. What about the others?

   As you can see, 4 1/5 is less than 4 4/5. Also note that 2/7 and 1/5 are both less than half, while 4/5 is more than half. So 4 4/5 is greater than either 4 2/7 or 4 1/5. But how do 4 2/7 and 4 1/5 compare with each other? They're close, so you can't tell just by looking. You can compare them if you find a common denominator.

   Because the whole-number parts of 4 1/5 and 4 2/7 are the same, you need to compare only the fractional parts, 2/7 and 1/5.

   The numbers 7 and 5 are both prime, so the common denominator is their product, 35.

   $\frac{2}{7} \cdot \frac{5}{5} = \frac{10}{35}$ and $\frac{1}{5} \cdot \frac{7}{7} = \frac{7}{35}$. Since $\frac{7}{35} < \frac{10}{35}$, it follows that $\frac{1}{5} < \frac{2}{7}$, so $4\frac{1}{5} < 4\frac{2}{7}$ and $21/5 < 30/7$.

   So the whole list, from least to greatest, is:
   3 6/7, 4 1/5, 4 2/7, 4 4/5

In the question's format, that's
$3\ 6/7,\ \ 21/5,\ 30/7,\ 4\ 4/5$

2. Write $22/7$ as a mixed number.

You can think of an improper fraction as a division problem. Carry out the division. The fraction $22/7$ equals 3 with a remainder of 1. The main part of the quotient (3 in this case) is the whole-number part of the mixed number, and the remainder part of the quotient is the fractional part of the mixed number, so the mixed number is $3\ 1/7$.

3. Simplify:
$\frac{22}{18} \div \frac{45}{10}$

Simplify here means *carry out the operation and then give your answer in simplest form*. In other words, divide.

To divide two fractions, multiply by the divisor's reciprocal:

$$\frac{22}{18} \div \frac{45}{10} = \frac{22}{18} \cdot \frac{10}{45}.$$

The next step is a prime factorization of both numerators and both denominators.

$$\frac{22}{18} \cdot \frac{10}{45} = \frac{2 \cdot 11}{2 \cdot 3 \cdot 3} \cdot \frac{2 \cdot 5}{3 \cdot 3 \cdot 5}.$$

The dot between the two fractions, meaning *multiply*, is not essential; since you multiply fractions by multiplying numerator by numerator and denominator by denominator, it may as well be just one numerator and just one denominator:

$$\frac{22}{18} \div \frac{45}{10} = \frac{2 \cdot 11 \cdot 2 \cdot 5}{2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 5}.$$

Rearrange so that you can see any factors equal to 1 -- that is, any number divided by itself -- and cancel those numbers out.

$$\frac{2 \cdot 11 \cdot 2 \cdot 5}{2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 5}. = \frac{2}{2} \cdot \frac{5}{5} \cdot \frac{11 \cdot 2}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{11 \cdot 2}{3 \cdot 3 \cdot 3 \cdot 3}$$

With no more to simplify, multiply the terms in the numerator to make a new numerator and multiply the terms in the denominator to make a new denominator, and you're done.

$$\frac{11 \cdot 2}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{22}{81}$$

The answer is $\frac{22}{81}$.

Notes:

1. You **do not need a common denominator to multiply or divide fractions**, only to add or subtract them. Using a common denominator for fraction multiplication or division will usually get you a lot of unnecessary work.
2. In the steps above, you simplify the fractions and then multiply them. It is equally valid to multiply first and then simplify -- but that's a lot more work.

4. Simplify:
$\frac{5}{6} + \frac{7}{18} \div \frac{3}{2}$

Again, *simplify* means *carry out the operation.* The tricky part this time is to do things in the right order; specifically, you need to do the division before the addition. **Multiplication and division come before addition and subtraction.**

You may want to use parentheses to show (and remind yourself of) the correct order:
$\frac{5}{6} + \left(\frac{7}{18} \div \frac{3}{2}\right)$

To do the division, multiply by the divisor's reciprocal:

$\frac{7}{18} \div \frac{3}{2} = \frac{7}{18} \cdot \frac{2}{3} = \frac{7}{2 \cdot 3 \cdot 3} \cdot \frac{2}{3}$.

Cancel the 2s and you get $\frac{7}{18} \div \frac{3}{2} = \frac{7}{3 \cdot 3} \cdot \frac{1}{3} = \frac{7}{27}$.

Going back to the larger problem:

$\frac{5}{6} + \frac{7}{18} \div \frac{3}{2} = \frac{5}{6} + \frac{7}{27}$.

Now you have to add the fractions. To do that you need a common denominator. The least common denominator contains every prime factor contained in either denominator, and it contains each factor the greatest number of times that factor is contained in either denominator.

What does that mean? Let's look at the denominators:

$6 = 2 \cdot 3$ and $27 = 3 \cdot 3 \cdot 3$. The factor 2 shows up just once in one of the denominators, and the factor 3 shows up once in the first denominator and three times in the second denominator. Then the least common denominator contains one power of 2 and three powers of $3.$ That's $2 \cdot 3 \cdot 3 \cdot 3 = 54.$

Convert both terms into fractions with a denominator of 54:
$\frac{5}{6} \cdot \frac{9}{9} = \frac{45}{54}$ and $\frac{7}{27} \cdot \frac{2}{2} = \frac{14}{54}$.

So the problem becomes:
$\frac{5}{6} + \frac{7}{18} \div \frac{3}{2} = \frac{45}{54} + \frac{14}{54} = \frac{59}{54}$

5. Trying to fix her broken computer while tech support guides her through the steps on the telephone, Samantha realizes that the $\frac{7}{16}$-inch screwdriver she is using is $\frac{1}{16}$ inch too small. What size screwdriver does Samantha need?

   Samantha needs a screwdriver whose size is equal to the size of the one she has plus the difference between the size she needs and the size she has. That's $\frac{7}{16} + \frac{1}{16} = \frac{8}{16}.$ But you are expected to give answers in simplified form.

   $\frac{8}{16} = \frac{1}{2}.$

6. Evaluate $1.6 \div .4$.

   Evaluate also means *carry out the operation.*
   You could express this division problem as a fraction: $1.6 \div .4 = \frac{1.6}{.4}$. To get the decimals out of $\frac{1.6}{.4}$, multiply the fraction by 1 in the form of $\frac{10}{10}$:
   $\frac{1.6}{.4} \cdot \frac{10}{10} = \frac{16}{4}$. And $\frac{16}{4} = 4.$

7. What is the decimal value of 120%?

   Remember that *percent* means *divided by 100*. So 120% means $120 \div 100.$ Expressed in decimal format, that's 1.2.

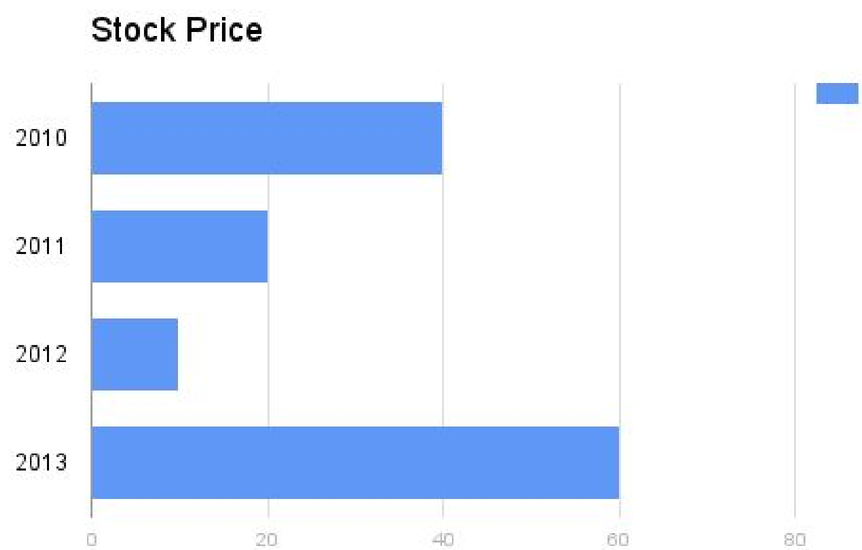
8. Jack bought pants for $26.50. There was a 6% sales tax on the pants. How much did the pants cost before sales tax?

   In word problems like this, half the challenge is to figure out what the problem is talking about. Was it $26.50 plus tax? Couldn't be, because the cost of the pants without tax is what you're being asked for -- so the cost you're told, $26.50, must be the cost including tax.

   To find the cost without tax, first give that amount a letter name, then put the information you know about it into an equation.

Call the cost without tax x. The total cost (that is, the cost including tax) is x plus tax. Tax is equal to 6% of x, or $.06x$, so the total cost equals $x + .06x$, or $1.06x$. The total cost also equals 26.50. Now you can write an equation: $1.06x = 26.50$. To solve, divide both sides by 1.06. Then $x = 25$. Jack's pants cost $25 plus tax.

9. Find the percent increase in stock price of the ABC fund from 2010 to 2013. Use the graph below.



You're asked for a comparison between the price for 2010 and the price for 2013. The other two prices in the graph are not relevant.

The percent increase is the increase divided by the original amount, expressed as a percent. That's $\frac{60-40}{40} = \frac{20}{40}$. Expressed as a percent, that's 50%.

10. How many pints are in 2 gallons?

There are 2 pints in a quart and 4 quarts in a gallon. That means there are $2 \times 4 = 8$ pints in 1 gallon, so there are $8 \times 2 = 16$ pints in 2 gallons.

11. Evaluate 4^3.

The expression 4^3 means 4 taken as a factor 3 times -- in other words, $4 \times 4 \times 4$. That's 64.

12. Evaluate $7-2\cdot(-3)^2$.

In this question, as in question 4, the order in which operations are carried out is important. Do them in the wrong sequence and you will almost always get the wrong answer.

Start with grouping symbols. This problem uses parentheses as grouping symbols. The first step in evaluating would be to simplify the contents of the parentheses -- but there's nothing to simplify.
Next, carry out any exponentiation in the problem. Here, -3 is taken to the second power. Let's mark that with brackets:
$7-2\cdot\left[(-3)^2\right]$

After that comes multiplication. Mark that with braces.
$7-\left\{2\cdot\left[(-3)^2\right]\right\}$

This problem contains just one multiplication. If a problem contains both multiplication and division, or more than one of either, start on the left and work your way right.

Next do addition and subtraction. This problem contains just one subtraction. If a problem contains both addition and subtraction, or more than one of each, start on the left and work your way right.

Now let's go back and carry out those operations. Start with the brackets:
$\left[(-3)^2\right]$

That's -3 taken as a factor twice: $-3\cdot-3$. That's 9. The next part is:
$\left\{2\cdot\left[(-3)^2\right]\right\}=2\cdot 9=18.$

Finally,
$7-\left\{2\cdot\left[(-3)^2\right]\right\}=7-18=-11.$

13. Simplify $n(3-p+n)+2(2p-n^2-1)$

Distribute. In other words, multiply the term outside the parentheses by the contents of the parentheses:

$n(3-p+n)=3n-np+n^2$ and $2(2p-n^2-1)=4p-2n^2-2.$ Then
$n(3-p+n)+2(2p-n^2-1)=3n-np+n^2+4p-2n^2-2.$

At this point, all that's left to do is to combine the two n^2 terms:

$$3n-np+n^2+4p-2n^2-2=3n-np-n^2+4p-2.$$

14. Solve for x: $0.25x=5.$

Since 0.25 is another way to say $1/4,$ multiply both sides of the equation by 4:

$$4\cdot 0.25x=4\cdot 5$$
$$x=20$$

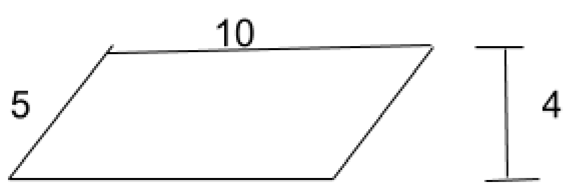
Alternatively, you could divide both sides of the equation by $0.25.$ That amounts to the same thing as multiplying by $4,$ but multiplying by 4 is easier.

15. If a bakery sells a dozen cookies for $3.00, what would the cost of 4 cookies be?

In this problem there is an assumption that the cost per cookie does not depend on the number of cookies you buy. Real life doesn't always work that way, but for this problem you need to make that assumption.

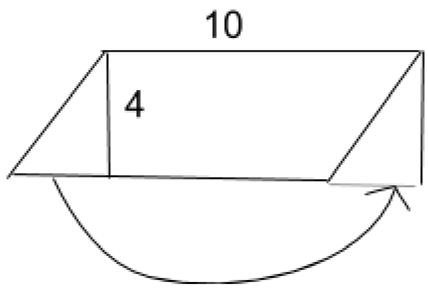
The cost per cookie is $3.00/12 cookies = $0.25 per cookie, so the cost of 4 cookies would be $4\cdot \$0.25=\$1.00.$

16. What is the area of the parallelogram?



There may be a temptation to multiply the 5 by the 10. That's how you find the area of a **rectangle**: by multiplying adjoining sides. Not so any old parallelogram. Think about it: If those acute angles were extremely acute, the whole thing would flatten down and the area would be clearly decreased even though the lengths of the sides did not change -- so multiplying the length of one side by the other just doesn't work here.

Then how **do** you find the area of a parallelogram?



What if you cut the triangle off the left-hand side of the parallelogram and then moved it to the right-hand side to form a rectangle? The area of the new rectangle would be its length $(10$, the parallelogram's base) times its width $(4,$ the parallelogram's height). And because nothing has been added or subtracted -- pieces have only been moved around -- the rectangle's area is the same as the parallelogram's. So the parallelogram's area is equal to the rectangle's length times its width, which is the same as the parallelogram's base times its height, $10 \times 4 = 40.$ **A parallelogram's area is equal to its base times its height.**

17. Solve for n: $2n + 4 = 5\,(2 - 2n)$.

$2n + 4 = 5\,(2 - 2n)$

$2n + 4 = 10 - 10n$ Distribute.

$12n = 6$ Add $10n$ to each side and subtract 4 from each side.

$n = 6/12 = 1/2$

18. Solve for x. $\frac{6x-6-3x}{8} = 9.$

$\frac{6x-6-3x}{8} = 9.$

$6x - 6 - 3x = 72$ Multiply both sides by 8.

$3x - 6 = 72$ Combine like terms.

$3x = 78$ Add 6 to each side.

$x = 26$ Divide both sides by 3.

19. Solve for R. $T = \frac{D}{R}$.

$T = \frac{D}{R}$

$RT = D$ Multiply both sides by R.

$R = \frac{D}{T}$ Divide both sides by T.

20. Solve the inequality and graph on a number line.

$8x + 5 \leq 2x - 1$

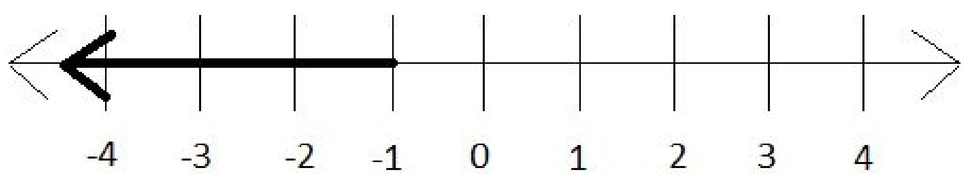
$8x + 5 \leq 2x - 1$

$6x + 5 \leq -1$ Subtract $2x$ from each side.

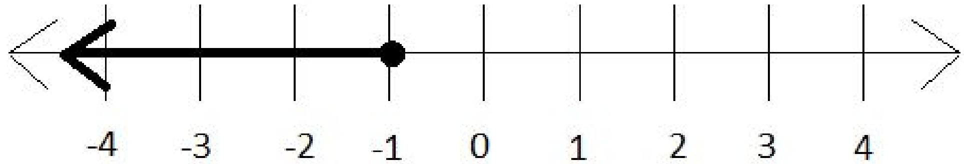
$6x \leq -6$ Subtract 5 from each side.

$x \leq -1$ Divide both sides by 6.

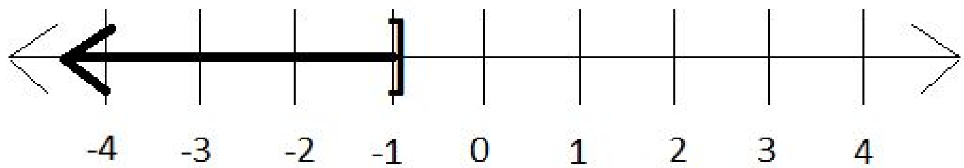
To denote all numbers less than or equal to -1 on a number line, start at -1 and draw an arrow to the left:



To show that -1 is included, you can use a filled-in circle:



An interval like this, in which the endpoints are included, is called *closed*. An alternative notation for the endpoint is a square bracket, as shown below.



21. Solve the inequality: $x - 4 \geq 3x + 8.$

$x - 4 \geq 3x + 8$	
$x \geq 3x + 12$	Add 4 to each side.
$-2x \geq 12$	Subtract $3x$ from each side.
$x \leq -6$	Divide both sides by $-2.$ Because you divide by $-2,$ you switch the direction of the inequality sign. **In an inequality, when you multiply or divide by a negative number, you switch the direction of the inequality sign.**

22. The perimeter of a rectangular garden is 58 feet. If the length of the garden is 1 foot less than 2 times the width, then what is the garden's length?

Since you know the length in terms of the width, give the width a letter name: $w.$ You're told the length is 1 foot less than twice the width. In terms of $w,$ that's $l = 2w - 1,$ where l is the garden's length. You're told, too, that the perimeter is 58 feet. The perimeter is 2 times the length plus 2 times the width. In terms of $w,$ that's $2(2w - 1) + 2w.$ The two expressions for the perimeter equal each other: $2(2w - 1) + 2w = 58.$

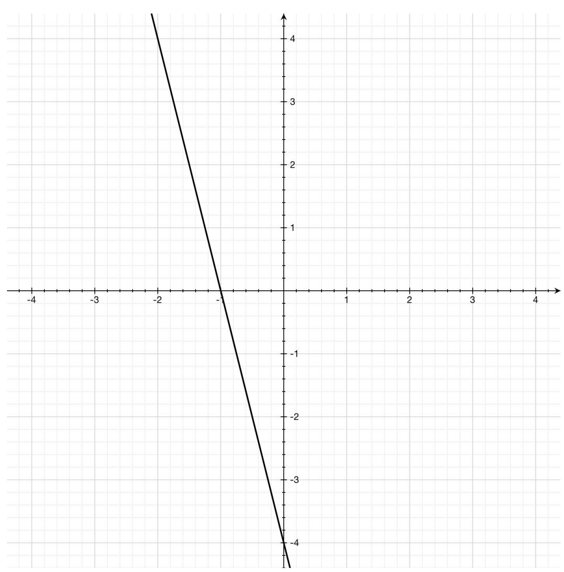
$2(2w - 1) + 2w = 58$	
$4w - 2 + 2w = 58$	Distribute.
$6w - 2 = 58$	Gather like terms.
$6w = 60$	Add 2 to each side.
$w = 10$	Divide both sides by 6.

$l = 2w - 1$	This is the expression you found for the length.
$l = 2(10) - 1$	Substitute the value of w that you found.
$l = 19$	Calculate the length.

23. Graph the equation $x = -\frac{1}{4}y - 1$.

This problem is tricky because it gives x in terms of y rather than what you are used to, y in terms of x. You can solve for y to get the equation into more familiar terms, or you can graph directly.

Let's try graphing directly. In this format, it's easy to see the x-intercept, the value that x takes when $y = 0$. That's -1. So one point on the line is $(-1,\ 0)$. The next easiest point to find is the $y-$intercept, the value that y takes when $x = 0$. Plug in 0 for x, do a little rearranging, and you get that $y = -4$ when $x = 0$; that means the point $(0,\ -4)$ is on the line. These two points are enough to graph the line. It looks like this:



What is the slope? Choose two points. The intercepts (the points $(-1,\ 0)$ and $(0,\ -4)$, the two points found above) are convenient.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Definition of slope: change in y divided by change in x.
	Say $(-1,\ 0)$ is point 1; then $(0,\ -4)$ is point 2.

$x_1 = -1$

$y_1 = 0$

$x_2 = 0$

$y_2 = -4$

$m = \frac{-4-0}{0-(-1)}$
$m = -4$

The slope is $-4.$

Now let's try the other method: Solve for y to get an equation of the form $y = mx + b.$ Then you can read the slope and y-intercept from the equation.

$x = -\frac{1}{4}y - 1$

$x + 1 = -\frac{1}{4}y$ Add 1 to each side.

$-4x - 4 = y$ Multiply both sides by $-4.$

$y = -4x - 4$

Switch the sides.

As you can see, the slope is -4 and the $y-$intercept is $(0, \ -4)$, the same as those found with the first method. Check.

24. Graph the inequality $x > -2y + 4.$

To graph an inequality, break the problem down into two steps:

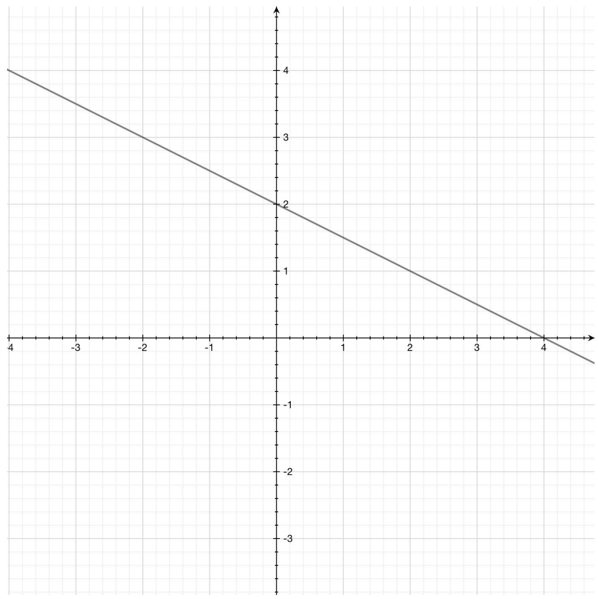
1. Graph the related equation.
2. Decide where the shading goes.

1. Graph the equation $x = -2y + 4.$

Because this equation gives x in terms of $y,$ the easiest way to graph it is the intercept method. That is, find the value of y when x is $0,$ then the value of x when y is $0,$ and use those two points to graph the equation.

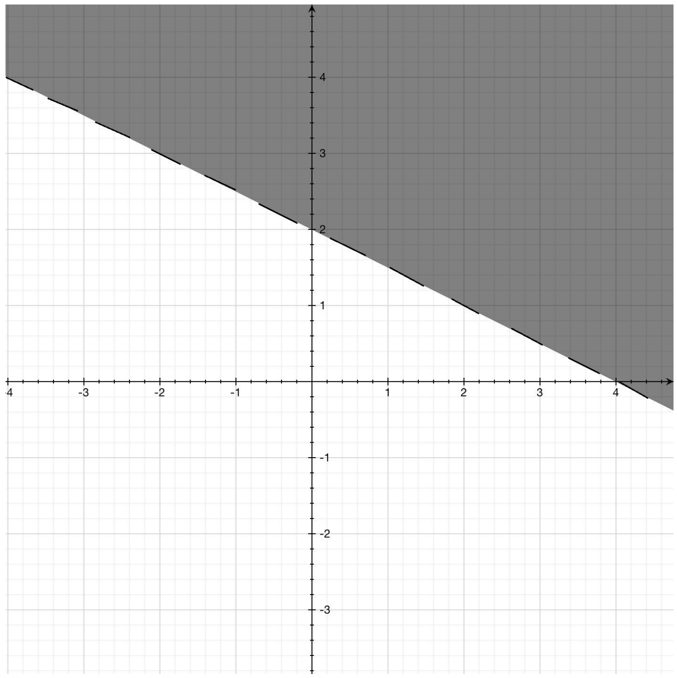
x	y
0	2
4	0

Graph the points $(0,\ 2)$ and $(4,\ 0)$ and connect them. The graph should look like this:



2. Shade the appropriate side. Which side is that?

Consider the inequality: $x > -2y + 4$. Saying x is greater means you are looking for the larger values of x. That is, the area that should be shaded is to the right of the line that represents the equation $x = -2y + 4$:



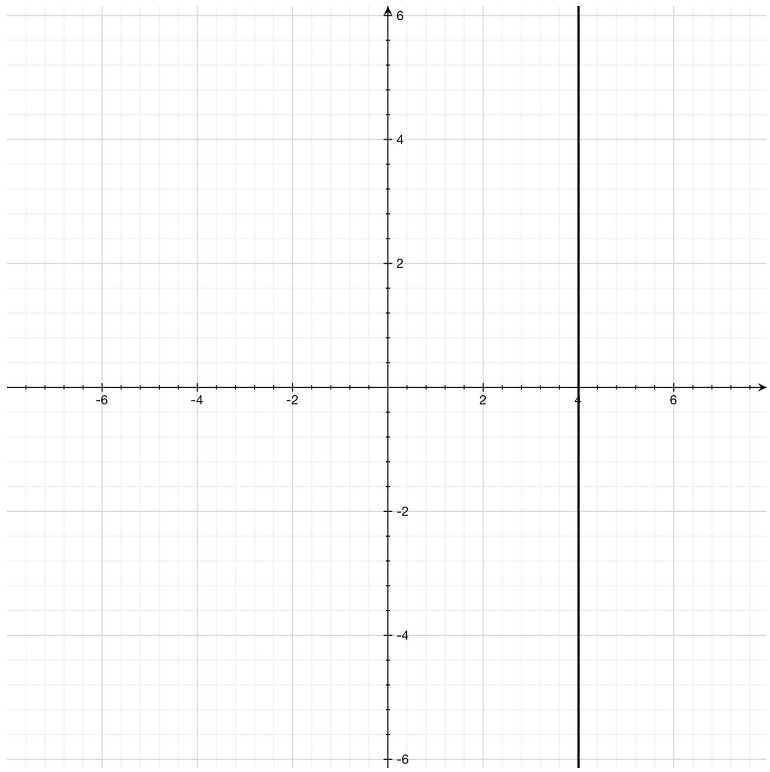
Use a dashed line, not a solid line, to represent the line, because the inequality is open -- that is, we're talking greater than, not greater than or equal to.

Another way to determine which side should be shaded is to test a point. If the inequality is true on that point, then the side of the inequality on which the test point falls gets shaded; if not, the other side gets shaded.
Usually the easiest point to test is the origin, $(0,\ 0)$. Let's try that with the inequality $x > -2y + 4.$ Substituting 0 for both x and y in the equality gives $0 > -2 \cdot 0 + 4,$ which simplifies to $0 > 4.$ True statement? No. Therefore the points in the area below the line and to its left do not satisfy the inequality and therefore do not get shaded. The points on the other side do. Shade the area above the line and to its right.

You can use either method to solve the problem and use the other method as a check.

25. What is the slope of the line that passes through the points $(4, -3)$ and $(4, 3)$?

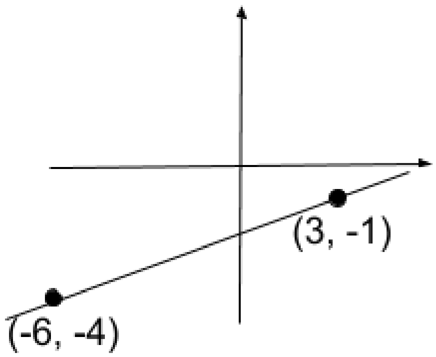
Graph the points to see what's going on:



Notice that the $x-$value is the same for both points: $4.$ On a straight line, if the $x-$value is the same for two points, it is the same for all points. The $y-$value, on the other hand, can be anything. The equation's line is $x = 4.$

Another way to look at this is to try to find the line's slope and $y-$intercept. The slope is the change in y divided by the change in x. But x isn't changing, so the change in $0.$ When you divide by $0,$ the result is undefined.. Therefore the slope is *undefined*.

26. A line contains the points $(3, -1)$ and $(-6, -4).$ What is the line's equation?



Graph the points and the line through them to get a sense of what the line looks like and therefore what its equation might be.

As you can see from the graph, the slope is positive and shallow (less than 1), and the $y-$intercept is negative.

Find the slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-4)}{3-(-6)} = \frac{3}{9} = \frac{1}{3}$.

Use the point-slope equation -- $y - y_1 = m(x - x_1)$ -- to find the $y-$intercept. The point (x_1, y_1) can be either of the points in the line; pick the one that looks easier. I'll choose $(3, -1)$ because it is the one with the smaller numbers and therefore looks easier.

$y - y_1 = m(x - x_1)$	
$y - (-1) = \frac{1}{3}(x - 3)$	For x_1 and y_1, substitute the coordinates of one point on the line.
$y + 1 = \frac{x}{3} - 1$ $y = \frac{x}{3} - 2$	Solve for y.

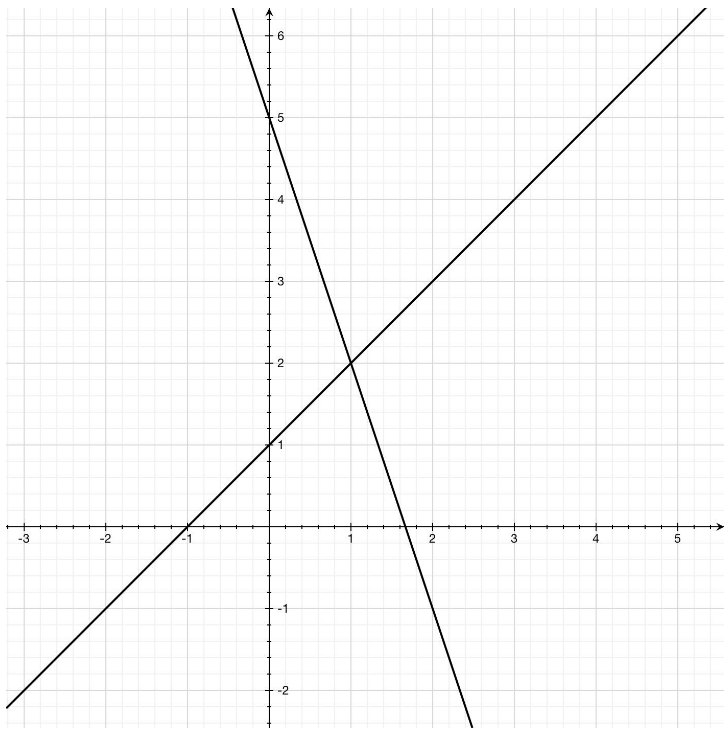
The equation is $y = \frac{x}{3} - 2$, which checks out against the initial information that the slope is positive and shallow and the $y-$intercept is negative.

For a better check, you can substitute the x-values for the points given with the problem into the equation you have found and see if you get the same y-value. For example, take the point $(-6, -4)$:

$y = \frac{x}{3} - 2$	
$y = \frac{-6}{3} - 2$	Substitute -6 for x.
$y = -4$	Solve for y.

Check.

27. Write the equations, in standard form, of the lines in the graph.



An equation in standard form looks like $ax + by = c$, where a and b are both integers and a is positive.

How can you find those equations? Probably the easiest way is to find each line's slope and intercept, write the equation in slope-intercept form, and then convert the equation to standard form.

Consider first the line that goes up as it goes to the right -- that is, the one with the positive slope. Read the line's intercepts from the graph: $(0,\ 1)$ and $(-1,\ 0.)$ The slope is the change in y divided by the change in x : $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{-1-0} = 1.$ The $y-$intercept is $(0,\ 1)$. So the equation is $y = x + 1.$

Now consider the other line. You can read the $y-$intercept from the graph: It's $(0,\ 5)$. But the $x-$intercept, a fraction, is harder to read. Instead, choose the point of intersection of the lines, $(1,\ 2)$. From these two points, find the line's slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{1-0} = -3.$ So this line's equation is $y = -3x + 5.$

The equations are $y = x + 1$ and $y = -3x + 5$. Convert them into standard form:

$y = x + 1$		$y = -3x + 5$	
$-x + y = 1$	Get both variables on one side, with the constant on the other side.	$3x + y = 5$	Get both variables on one side, with the constant on the other side.

$x - y = -1$ Multiply both sides by -1 so that the equation starts with a positive number.

The equations are $x - y = -1$ and $3x + y = 5$.