VPT Math Practice Problems
Units 6 - 9

1. Simplify $\frac{(3^2 5^2 7^1)^2}{3^4}$

Start with the numerator. Two rules can help you simplify this:

- $(ab)^c = a^c b^c$ and
- $(a^b)^c = a^{bc}$.

So $\left(3^2 5^2 7^1\right)^2 = \left(3^2\right)^2 \cdot \left(5^2\right)^2 \cdot \left(7^1\right)^2 = 3^4 5^4 7^2$. Substitute that for the numerator in the original expression:

$$\frac{(3^2 5^2 7^1)^2}{3^4} = \frac{3^4 5^4 7^2}{3^4}$$

The factors of 3 are the same in both numerator and denominator, so they cancel. $\frac{3^4 5^4 7^2}{3^4} = 5^4 7^2$. Note that you don't have to multiply out 3^4 . Just cancel it.

Now multiply out what's left. $5^4 7^2 = 30,625$

2. Simplify $2xy + (3x + 4y)(5x + xy + z - 4y)$.

First multiply together the two terms in parentheses, then add the first term to that product.

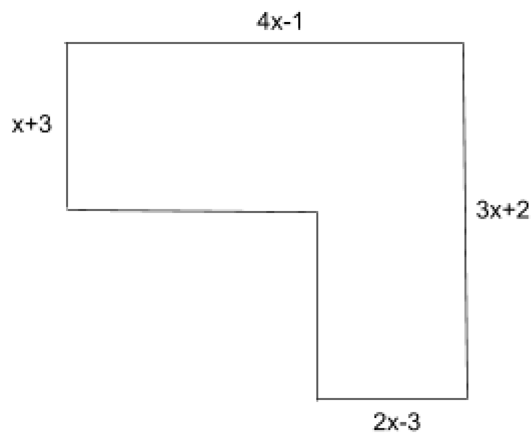
To multiply one sum by another you multiply each term in one set of parentheses by each term in the other. This is like FOIL, but more complicated because the second set of parentheses contains four terms, not just two.

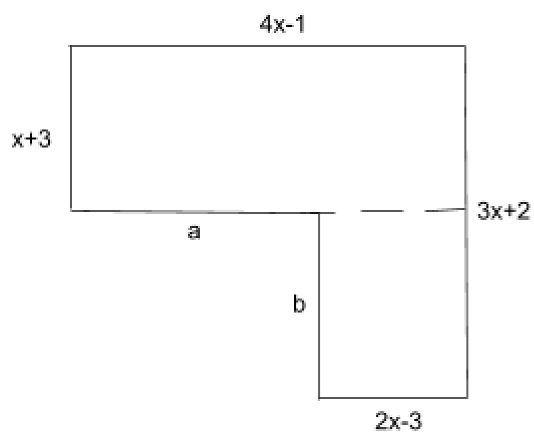
To do that, multiply $3x$ by $5x$ to get $15x^2$. Then multiply $3x$ by xy to get $3x^2y$. Keep going like that until you've multiplied $3x$ by every term in the second set of parentheses. The result should be $15x^2 + 3x^2y + 3xz - 12xy$. Now multiply $4y$ by each term in the second set of parentheses. That result should be $20xy + 4xy^2 + 4yz - 16y^2$. Now add the two sets of results, combining like terms. There is only one set of like terms in these two collections of terms: $-12xy$ and $20xy$. The sum of those two terms is $8xy$. Then the whole sum is $15x^2 + 3x^2y + 3xz + 8xy + 4xy^2 + 4yz - 16y^2$.

Now add $2xy$. The term $2xy$ is like $8xy$, so the two terms $2xy$ and $8xy$ can be combined into $10xy$ and the final answer is $15x^2 + 3x^2y + 3xz + 10xy + 4xy^2 + 4yz - 16y^2$.

3. What is the area of the figure shown?

To find the figure's area, you need to assume that all the angles in the figure that look like right angles are in fact right angles. Once you make that assumption, you can break the figure up into two rectangles:



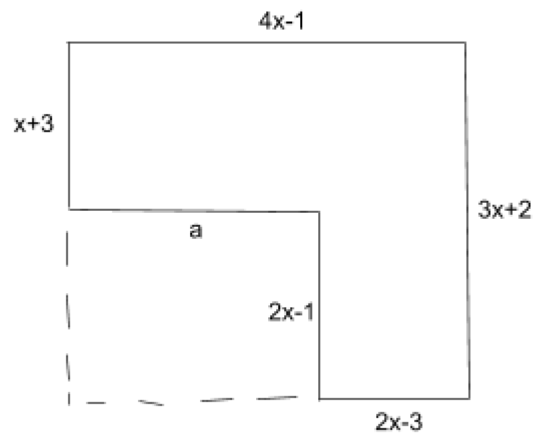


The rectangle on top has dimensions $4x - 1$ and $x + 3$, so its area is $(4x - 1)(x + 3) = 4x^2 + 11x - 3$.

What about the other piece? You can see that its base is $2x - 3$. What about its height, labeled b? That must be the difference between the height of the right-hand side, $3x + 2$, and the height of the far-left-hand side, $x + 3$. That would make b equal to $3x + 2 - (x + 3) = 2x - 1$. Then the area of the bottom piece is

$(2x-3)(2x-1) = 4x^2 - 8x + 3$. The total area is the sum of the areas of the two pieces: $4x^2 + 11x - 3 + 4x^2 - 8x + 3 = 8x^2 + 3x$. That's the answer.

As a check, you can calculate the area a different way. Think of the shape as one big rectangle with a chunk missing. Find the area of the big rectangle and subtract the area of the chunk.



The big rectangle's area is $(4x-1)(3x+2) = 12x^2 + 5x - 2$. To find the area of the small rectangle you need to know not just b, calculated above as $2x-1$, but also a. That length must be the difference between the top and the bottom, $4x - 1 - (2x-3) = 2x+2$. So the area of the small rectangle is $(2x+2)(2x-1) = 4x^2 + 2x - 2$. Subtract that from the area of the big rectangle to find the area of the figure you were given: $12x^2 + 5x - 2 - (4x^2 + 2x - 2) = 8x^2 + 3x$. Same answer. Check.

4. Factor $6x^2 - xy - 12y^2$.

To start, look for a common factor. There is none.

Then use the a-c method. Multiply the coefficient of the first term, 6, by the coefficient of the last term, -12. That gives you -72. Find factors of -72 that add up to the coefficient of the middle term, -1.

To find those numbers, consider the factors of -72: $-1 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$. Notice that -72 is a negative number, and that the sum of the numbers you are looking for is equal to -1. That means the two numbers will be one positive number and one negative number, and the negative number's absolute value is 1 more than the positive number's -- that's how you get -1 when you add them. What factors of 72 differ by 1? 8 and 9. The numbers you seek are -9 and 8. Now that you've got those two numbers, you can factor by grouping:

$6x^2 - xy - 12y^2$	
$6x^2 - 9xy + 8xy - 12y^2$	Rewrite the middle term in two pieces whose coefficients are the numbers you found, -9 and 8.
$3x(2x-3y) + 4y(2x-3y)$	Factor out the common factor in the first two terms, as well as the common factor in the last two terms.

The first two terms have a common factor of $3x$; the last two have a common factor of 4.

Make sure the contents of the two sets of parentheses are the same. In this case they are, $2x-3y$.

$(3x+4y)(2x-3y)$	Complete the factoring. This is the final answer.
$6x^2-xy-12y^2$	FOIL back to check.

Note: The actual practice test question gives you a choice of binomials and asks you to find the one that is a factor of a trinomial that it gives you. You could solve this by factoring the trinomial, as shown above. Or you could do polynomial long division to try the various answer choices until you find the one that works. (That is, divide $6x^2-xy-12y^2$ by each factor until you find one that gives a remainder of 0. That binomial is a factor of $6x^2-xy-12y^2$.)

How to do polynomial long division:

- Set the problem up as long division.

$$3x+4y\,\overline{)\,6x^2-xy-12y^2}$$

- Divide the first term of the quantity being divided -- in this case $6x^2$ -- by the first term of the divisor and write the result above the line, near the left-hand end.

$$\begin{array}{r}2x\\ 3x+4y\,\overline{)\,6x^2-xy-12y^2}\end{array}$$

- Multiply what you just wrote by the divisor. In this case that's $2x(3x+4y)$. Write the result under the first two terms of the divisor.

$$\begin{array}{rl} & 2x \\ 3x+4y & \overline{)\,6x^2-xy-12y^2} \\ & 6x^2+8xy \end{array}$$

- Set this up as a subtraction. You are going to subtract the binomial you just found from the first terms of the divisor.

$$\begin{array}{rl} & 2x \\ 3x+4y & \overline{)\,6x^2-xy-12y^2} \\ & \underline{-(6x^2+8xy)} \end{array}$$

- Carry out the subtraction.

$$
\begin{array}{r}
2x \\
3x+4y\,\big)\,\overline{6x^2-xy-12y^2} \\
-(6x^2+8xy) \\
\hline
-9xy-12y^2
\end{array}
$$

- Divide the first term of the difference by the first term of the divisor. In this case that's $-9xy/(3x+4y)$. Subtract that result from the two terms on the line above.

$$
\begin{array}{r}
2x-3y \\
3x+4y\,\big)\,\overline{6x^2-xy-12y^2} \\
-(6x^2+8xy) \\
\hline
-9xy-12y^2 \\
-(-9xy-12y^2) \\
\hline
0
\end{array}
$$

The result is zero. That means $3x+4y$ is a factor of $6x^2-xy-12y^2$, and the other factor is $2x-3y$.

If you use this method, you may have to try all four binomials to the right one.

5. Factor completely: $12x^2-11x-56$
   - A. $(4x-7)(3x-8)$
   - B. $(4x-7)(3x+8)$
   - C. $(4x+7)(3x-8)$
   - D. $(4x+7)(3x+8)$

Here are two possible approaches:

One approach

Use the a-c method: Multiply the coefficient of the first term, 12, by the last term, 56. That gets you 672, an unwieldy number that you will have to factor. So try a different approach: Instead of carrying out the multiplication, consider the prime factors of $(12)(-56)$: -1, 2, 2, 2, 2, 2, 3, 7. Combine those factors into two numbers that add up to -11.

How can you narrow down the possibilities? Notice that -11 is an odd number, so those two numbers that you add together to get them must include one even and one odd. (If you add two evens you get an even. If you add two odds, you get an even. To get an odd, you need to add one even and one odd.) The number 2 is not a factor of any odd number, so all those 2s -- which multiply out to 32 -- must be factors of just one of the two numbers you seek. Setting -1 aside for a moment, it's reasonable to guess that the other two factors are in the other number. That's $3 \cdot 7 = 21$. The number with the greater absolute value must be negative, so the two numbers you need may be -32 and 21. $-32 + 21 = -11$. That works. Use those two numbers, -32 and 21, to factor the trinomial:

$12x^2 - 11x - 56$	
$12x^2 - 32x + 21x - 56$	Break up the middle term. Rewrite the expression using the numbers you found, -32 and 21.
$4x(3x - 8) + 7(3x - 8)$	Factor out the common factor in the first two terms, as well as the common factor in the last two terms: Make sure the contents of the two sets of parentheses are the same. In this case they are, $3x - 8$.
$(4x + 7)(3x - 8)$	Complete the factoring. This is the final answer: choice C.
$12x^2 - 11x - 56$	FOIL back to check.

Another Approach

Since each answer choice for this question gives both factors, you can multiply back to see if the factors work. You could go through the factor pairs one by one, but it would be easier to eliminate some first. The last term, -56, is the product of the second terms of the two binomials. It is negative, so the last terms of the two binomials must be one positive and one negative. That rules out answer choices A and D.

B and C are left. FOIL them and see what happens:

B: $(4x-7)(3x+8)=12x^2+11x-56.$ Not the original trinomial.
C: $(4x+7)(3x-8)=12x^2-11x-56.$ That works.

So the answer is $(4x+7)(3x-8)$.

6. Find the solutions to the equation:

$6x^2+3x-18=0$

$3\,(2x^2+x-6)=0$	Notice that each of the three terms on the left-hand side is divisible by 3. Factor out the 3.
$2x^2+x-6=0$	Divide both sides by 3. Note that the right-hand side does not change when you divide it by 3.
$(2)\,(-6)=-12$	Find the product of the *a* and *c* terms.
$4,\ -3$	Find two numbers that add up to the *b* term, 1, and multiply to -12.
$2x^2+4x-3x-6=0$	Rewrite the left-hand side of the equation with its middle term broken into two pieces whose coefficients are the numbers you just found.
$2x\,(x+2)-3\,(x+2)=0$	Factor out common factors from each pair of terms: the first and second terms, and the third and fourth terms.
$(2x-3)\,(x+2)=0$	Complete the factoring.
	Set each factor separately equal to 0, and solve.

$2x-3=0$ $\qquad$ $x+2=0$

$x=3/2$ $\qquad$ $x=-2$

These are the final answers.

There are two answers: $x = -3/2$ and $x = 2$. Substitute these numbers back into the original equation to check.

7. What value of x, if any, makes the following expression undefined?

$$\frac{x^5y^3}{25x^3y^2}$$

An expression is undefined if its denominator is equal to 0.

If $x = 0$ then the expression's denominator is equal to 0, and the expression is undefined. This is true even though for the simplified form of this expression, $\frac{x^2y}{25}$, no value of x will cause the denominator to equal 0.

Answer: $x = 0$

8. Simplify

$$\frac{2x^2+7x+5}{x^2+3x+2}$$

Look for factors that are common to the numerator and the denominator. To find such factors, factor the numerator and the denominator.

$2x^2 + 7x + 5 = (2x + 5)(x + 1)$ See questions 5 and 6 for detailed instructions on how to factor a trinomial.

$x^2 + 3x + 2 = (x + 2)(x + 1)$

Then

$$\frac{2x^2+7x+5}{x^2+3x+2} = \frac{(2x+5)(x+1)}{(x+2)(x+1)} = \frac{2x+5}{x+2}$$

Here's a little cheat: Any factor that you'll be able to cancel out must be present in both the numerator and the denominator. You can see that the denominator is easier to factor than the numerator. To find a factor that may be present in both, you can factor the denominator into two binomials and then try to divide the numerator by each of those binomials. This is more work than just factoring the numerator and the denominator separately, but it's something to keep in mind if you get stuck trying to factor the numerator.

9. Simplify

$$\frac{3x^2-11x-20}{x^2-3x-10}$$

Same general procedure as #8.

Numerator. Multiply the *a* and *c* terms: $3 \cdot -20 = -60$. Find the prime factors of -60: $-1,\ 2,\ 2,\ 3, 5.$ Your challenge now is to recombine those numbers into two numbers that add up to -11.

Note that -11 is odd, and the list of factors includes two factors of 2. Pairs of numbers that add up to an odd number include one even number and one odd number. (Even plus even equals even; odd plus odd equals odd; and odd plus even or even plus odd equals odd.) So the two numbers you need must include one even and one odd. One factor of 2 is enough to make a number even, so since one of the numbers must be odd, both factors of 2 must be in the same number. Perhaps one of the two numbers is $2 \cdot 2 = 4.$ Then the other number must contain the remaining factors, $-1,\ 3,\ 5.$ The product of those numbers, $-1 \cdot 3 \cdot 5 = -15.$ The two numbers 4 and -15 add up to -11. Those must be the two numbers you want.

Now factor by grouping. For more info on factoring by grouping, see questions 5 and 6.

$$3x^2 - 11x - 20$$
$$3x^2 - 15x + 4x - 20$$
$$3x(x - 5) + 4(x - 5)$$
$$(3x + 4)(x - 5)$$

Multiply back to check.

Denominator.

$$x^2 - 3x - 10 = (x - 5)(x + 2)$$

Then

$$\frac{3x^2-11x-20}{x^2-3x-10} = \frac{(3x+4)(x-5)}{(x-5)(x+2)} = \frac{3x+4}{x+2}$$

$\frac{3x+4}{x+2}$ is the final answer.

If factoring the numerator seems like too much, see the cheat suggested at the end of question 8.

10. Simplify

$$\frac{2x}{x-1} + \frac{3}{x+2}$$

To combine expressions that have different denominators, you need to convert them to a common denominator. Since the two denominators, $x-1$ and $x+2$, have no common factors, the common denominator is the product of the two. To convert $\frac{2x}{x-1}$, multiply by 1 in the form of $\frac{x+2}{x+2}$: $\frac{2x}{x-1} \cdot \frac{x+2}{x+2} = \frac{2x^2+4x}{x^2+x-2}$

Similarly, $\frac{3}{x+2} \cdot \frac{x-1}{x-1} = \frac{3x-3}{x^2+x-2}$

Then $\frac{2x}{x-1} + \frac{3}{x+2} = \frac{2x^2+4x}{x^2+x-2} + \frac{3x-3}{x^2+x-2} = \frac{2x^2+7x-3}{x^2+x-2}$

At this point you need to check again for common factors between the numerator and the denominator. But the numerator doesn't factor (you can see that because there are no integer factors of -6 that add up to 7), so there are none.

11. Simplify

$$\frac{6x+2}{x^2+5x+6} \div \frac{9x+3}{x^2+6x+8} \cdot \frac{x^2-x-12}{x+4}$$

Remember that to divide two rational expressions, you multiply by the divisor's reciprocal. So flip the second rational expression:

$$\frac{6x+2}{x^2+5x+6} \div \frac{9x+3}{x^2+6x+8} \cdot \frac{x^2-x-12}{x+4} = \frac{6x+2}{x^2+5x+6} \cdot \frac{x^2+6x+8}{9x+3} \cdot \frac{x^2-x-12}{x+4}$$

Next, factor wherever you can:

$$\frac{6x+2}{x^2+5x+6} \cdot \frac{x^2+6x+8}{9x+3} \cdot \frac{x^2-x-12}{x+4} = \frac{2(3x+1)}{(x+2)(x+3)} \cdot \frac{(x+2)(x+4)}{3(3x+1)} \cdot \frac{(x+3)(x-4)}{(x+4)}$$

Then cancel any factors that are common between the numerator and the denominator:

$$\frac{2(3x+1)}{(x+2)(x+3)} \cdot \frac{(x+2)(x+4)}{3(3x+1)} \cdot \frac{(x+3)(x-4)}{(x+4)} = \frac{2(3x+1)(x+2)(x+4)(x+3)(x-4)}{3(3x+1)(x+2)(x+4)(x+3)} = \frac{2(x-4)}{3}$$

12. Solve for *x*.

$\frac{3x}{x+1} = 1\frac{1}{2}$

A mixed number is difficult to work with, so convert the right-hand side into an improper fraction:

$1\frac{1}{2} = \frac{3}{2}$

Now the equation is

$\frac{3x}{x+1} = \frac{3}{2}$

Notice that both sides are multiples of 3, and you can simplify the problem by dividing both sides by 3. This step is not essential; it just makes the numbers easier.

$\frac{x}{x+1} = \frac{1}{2}$

Using the multiplication property of equality, multiply both sides by both denominators. That is, multiply both sides by $2 \cdot (x+1)$. This will clear the fractions.

$2 \cdot (x+1)\,\frac{x}{x+1} = 2 \cdot (x+1)\,\frac{1}{2}$

On each side, cancel terms that show up in both the numerator and the denominator.

$2x = x + 1$

Solve:

$x = 1$ That's the answer. To check, substitute it back into the original equation.

13. Which of the following is NOT equivalent to the expression $x^{\frac{1}{3}} \cdot x^{\frac{1}{3}} \cdot x^{\frac{1}{3}} \cdot x^{\frac{1}{3}}$?

A. $(\sqrt[3]{x})^4$

B. $\sqrt[3]{x^4}$

C. $\sqrt[3]{x^2} \cdot \sqrt[3]{x^2}$

D. $\left(\sqrt[3]{x^2}\right)^4$

Simplify the original expression. To multiply exponential terms that have the same base, add the exponents. $x^{\frac{1}{3}} \cdot x^{\frac{1}{3}} \cdot x^{\frac{1}{3}} \cdot x^{\frac{1}{3}} = x^{\frac{4}{3}}$. Which of the answer choices is not equivalent to that?

The question uses exponential format and the answer choices use radical format. Expressions in the same format are easier to compare than are expressions in different formats, so convert the answer choices into exponential format, which is the easier format to work with.

$$(\sqrt[3]{x})^4 = \left(x^{\frac{1}{3}}\right)^4 = x^{\frac{4}{3}}$$

$$\sqrt[3]{x^4} = (x^4)^{\frac{1}{3}} = x^{\frac{4}{3}}$$

$$\sqrt[3]{x^2} \cdot \sqrt[3]{x^2} = x^{\frac{2}{3}} \cdot x^{\frac{2}{3}} = x^{\frac{4}{3}}$$

$$\left(\sqrt[3]{x^2}\right)^4 = \left((x^2)^{\frac{1}{3}}\right)^4 = x^{\frac{8}{3}}$$

The last answer choice is not equal to the original expression.

14. Simplify

$$\sqrt[3]{640} \cdot \sqrt[3]{540}$$

There is a rule that $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ for real numbers a and b. So $\sqrt[3]{640} \cdot \sqrt[3]{540} = \sqrt[3]{640 \cdot 540}$. Any factor that occurs under the radical three times can be taken out from the radical. Factor both 540 and 640 to see what factors you can remove.

$$640 = 2^7 \cdot 5$$

$$540 = 2^2 \cdot 3^3 \cdot 5$$

$$640 \cdot 540 = 2^9 \cdot 3^3 \cdot 5^2$$

$$\sqrt[3]{640 \cdot 540} = \sqrt[3]{2^9 \cdot 3^3 \cdot 5^2}$$

The cube root of something cubed is just that something. In other words, $\sqrt[3]{a^3} = a$. You can take perfect cubes out of the radical. Start with 2:

$$\sqrt[3]{2^9} = \sqrt[3]{\left(2^3\right)^3} = \sqrt[3]{8^3} = 8$$

Similarly, $\sqrt[3]{3^3} = 3$. So $\sqrt[3]{2^9 \cdot 3^3 \cdot 5^2} = 8 \cdot 3\sqrt[3]{5^2} = 24\sqrt[3]{25}$

15. Simplify the expression using only positive exponents.

$$\frac{\left(m^{\frac{1}{4}}n^{\frac{3}{4}}\right)^3}{\left(m^{-\frac{1}{2}}n^{\frac{1}{4}}\right)^2}$$

Combine the exponents outside each set of parentheses with those inside per the rule $(a^b)^c = a^{bc}$. For the numerator, that means

$$\left(m^{\frac{1}{4}}n^{\frac{3}{4}}\right)^3 = m^{\frac{3}{4}}n^{\frac{9}{4}}$$

And applied to the denominator it means

$$\left(m^{-\frac{1}{2}}n^{\frac{1}{4}}\right)^2 = m^{-1}n^{\frac{2}{4}}$$

Then $\dfrac{\left(m^{\frac{1}{4}}n^{\frac{3}{4}}\right)^3}{\left(m^{-\frac{1}{2}}n^{\frac{1}{4}}\right)^2} = \dfrac{m^{\frac{3}{4}}n^{\frac{9}{4}}}{m^{-1}n^{\frac{2}{4}}}$

Remember the quotient rule: $\dfrac{m^a}{m^b} = m^{a-b}$

Then $\dfrac{m^{\frac{3}{4}}}{m^{-1}} = m^{\frac{3}{4}-(-1)} = m^{\frac{7}{4}}$

And $\dfrac{n^{\frac{9}{4}}}{n^{\frac{2}{4}}} = n^{\left(\frac{9}{4}-\frac{2}{4}\right)} = n^{\frac{7}{4}}$

So $\dfrac{m^{\frac{3}{4}}n^{\frac{9}{4}}}{m^{-1}n^{\frac{1}{2}}} = m^{\frac{3}{4}-(-1)}n^{\frac{9}{4}-\frac{2}{4}} = m^{\frac{7}{4}}n^{\frac{7}{4}}$

16. Perform the indicated operation and simplify.

$$\sqrt{27} + 2\sqrt{3}$$

You can add the two terms once you make them into like terms. To make the terms into like terms, simplify them if possible.

$\sqrt{27} = \sqrt{3^2 \cdot 3} = 3\sqrt{3}$

$\sqrt{3}$ cannot be simplified.

Then $\sqrt{27} + 2\sqrt{3} = 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}.$

17. Rationalize the denominator and simplify.

$$\frac{2\sqrt{3}}{\sqrt{15}}$$

Simplify first, then rationalize, then simplify again if necessary.

Break the radical in the denominator into its factors: $\sqrt{15} = \sqrt{3 \cdot 5} = \sqrt{3} \cdot \sqrt{5}.$

Then $\frac{2\sqrt{3}}{\sqrt{15}} = \frac{2\sqrt{3}}{\sqrt{3} \cdot \sqrt{5}}.$

Cancel the factor of $\sqrt{3},$ which is common to the numerator and the denominator:

$\frac{2\sqrt{3}}{\sqrt{15}} = \frac{2\sqrt{3}}{\sqrt{3} \cdot \sqrt{5}} = \frac{2}{\sqrt{5}}$.

To rationalize the denominator, multiply by $\frac{\sqrt{5}}{\sqrt{5}}$:

$\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{2\sqrt{5}}{5}$. $\frac{2\sqrt{5}}{5}$ is the final answer.

Alternatively, you could just dive in and rationalize the denominator in the original expression, without simplifying first. To do that, multiply $\frac{2\sqrt{3}}{\sqrt{15}}$ by $\frac{\sqrt{15}}{\sqrt{15}}$.

$$\frac{2\sqrt{3}}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{2\sqrt{45}}{15}.$$

Since $\sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$, $\frac{2\sqrt{45}}{15} = \frac{2 \cdot 3\sqrt{5}}{15} = \frac{2\sqrt{5}}{5}$

Same answer.

18. Solve.

$\sqrt{x+2} + \sqrt{x} = 2$

The variable is under the radical sign, so you need to square the expression to be able to work with the variable. But if you square both sides of the equation as it is, the left-hand side will still contain a radical and it will be pretty messy. To deal with that, start by subtracting $\sqrt{x}$ from both sides:
$\sqrt{x+2} = 2 - \sqrt{x}$

Now square both sides:

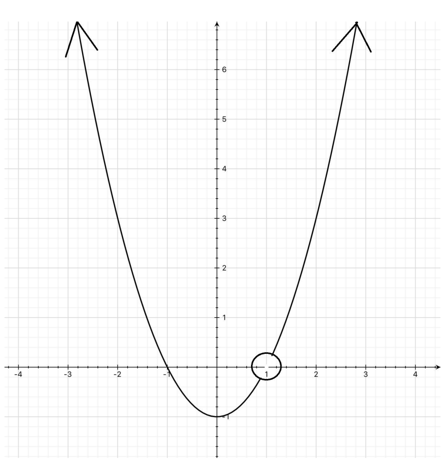
$x + 2 = 4 - 4\sqrt{x} + x$

You still have a radical on the right-hand side. Isolate the radical term.
$-2 = -4\sqrt{x}$
$\frac{1}{2} = \sqrt{x}$

Square both sides again to dump the radical.
$x = \frac{1}{4}$. That's the answer.

To check, substitute back into the original equation.

19. Use the graph to answer the question.

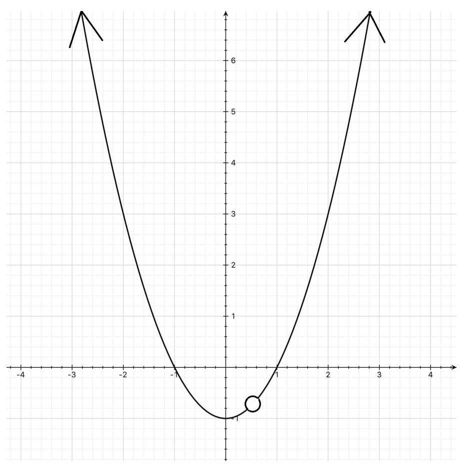


Is the graphed relation a function? If so, what is its domain?

Yes, the graphed relation is a function. You can see that because for every value of x there is only one value of y.

The domain is all real numbers except any that are shown to be excluded. The open circle at the point (1, 0) shows that that point is not included. So the domain is all real numbers except 1.

20. Use the graph to answer the question.



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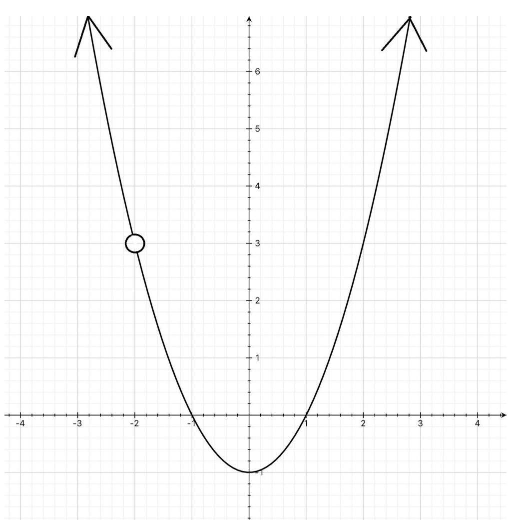
The domain is all real numbers except any that are shown to be excluded. The open circle at the point where $x = 1/2$ shows that that point is not included. So the domain is all real numbers except $1/2$.

21. Use the graph to answer the question.



Is the graphed relation a function? If so, what is its domain?

Yes, the graphed relation is a function. You can see that because for every value of x there is only one value of y.

The domain is all real numbers except any that are shown to be excluded. The open circle at the point where $x = -2$ shows that that point is not included. So the domain is all real numbers except -2.

22. Use the quadratic formula to solve the equation.

$x^2 + 10x + 6 = 0$

The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where a, b, and c are elements of a quadratic equation as follows: $ax^2 + bx + c = 0$.

For the given equation, $a = 1$, $b = 10$, and $c = 6$. Then

$x = \frac{-10 \pm \sqrt{100 - 4 \cdot 1 \cdot 6}}{2 \cdot 1}$

$x = \frac{-10 \pm \sqrt{76}}{2}$

$x = \frac{-10 \pm 2\sqrt{19}}{2}$

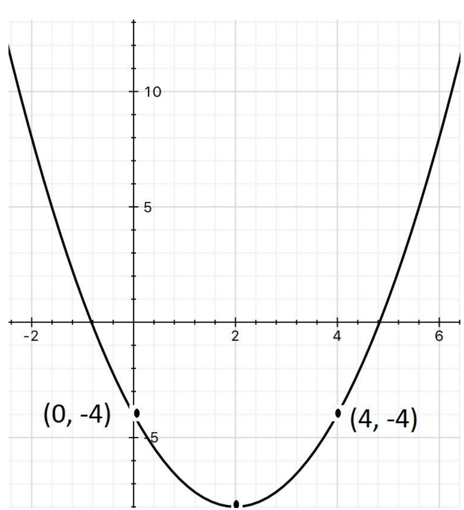
That comes out to $x = -5 + \sqrt{19}$ and $x = -5 - \sqrt{19}$.

23. Graph the equation $y = x^2 - 4x - 4$

The first line of attack for graphing a quadratic equation (like this one) should be to try to factor it. Some can be factored, others cannot. Can you find two binomials $(x + a)(x + b)$ that multiply out to $x^2 - 4x - 4$? The numbers *a* and *b* must be one positive and one negative, since their product, -4, is negative. The sum of those numbers must add up to the middle term's coefficient, -4. The only integer factor pairs of -4 are 1, -4; -1, 4; and -2, 2. None of these pairs has a sum of -4. So no, this expression can't be factored.

Plan B is to find points you can graph. The easiest point is the y-intercept. If $x = 0$, $y = -4$, so the graph goes through the point $(0, -4)$.

Next, look for the vertex. The vertex's x-value is at $-\frac{b}{2a} = -\frac{-4}{2 \cdot 1} = 2$. Plug that value, 2, into the equation to find the *y*-value at the vertex: $y = 2^2 - 4 \cdot 2 - 4 = -8$. The vertex is at $(2, -8)$.



This is a parabola. It is symmetric about its axis. (The parabola's axis is a vertical line that pases through the vertex. For this parabola, the equation of that line is $x = 2$.) That means two points at equal distances on opposite sides of the parabola's axis will have the same *y*-value. The y-intercept, 2 units to the left of the axis, has a *y*-value of -4.

Then the point 2 units to the right of the axis also has a *y*-value of -4; the point is $(4,\ -4)$.

These 3 points -- $(0,\ -4)$, $(2,\ -8)$, and $(4,\ -4)$ -- are enough for you to graph the parabola approximately. If you want to include more points, you can plug any value of *x* into the equation to find the corresponding *y*-value, or you can use the quadratic formula to find the *x*-intercepts.

24. A paper airplane is thrown from the top of a 200-foot cliff. The height of the airplane at any time is given by the formula $h = -3t^2 + 200$, where *h* is the airplane's height at *t* seconds. How many seconds after the airplane is thrown will its height be 109.25 feet?

You are given the equation $h = -3t^2 + 200$. You are told a height*, h,* and you need to solve for *a* time, *t*. Substitute the value of *h* you are given into the equation:

$109.25 = -3t^2 + 200$

Solve for *t*:

$109.25 = -3t^2 + 200$

$3t^2 = 90.75$	Add $3t^2$ to, and subtract 109.25 from, both sides of the equation.
$t^2 = 30.25$	Divide both sides by 3.
$t = 5.5$	Find the square root of both sides.

The airplane will be 109.25 feet above the ground **5.5 seconds** after it is thrown.

Note that there is an assumption that time $t = 0$ is the time when the airplane is thrown. In this problem you do not deal with negative values of *t*, because *t* is negative before the airplane is thrown.