Rational Functions

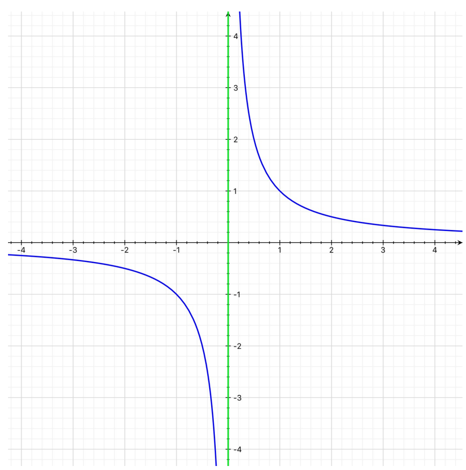
1 Introduction

A *rational function* is a fraction with variables in its denominator, and usually in its numerator as well.

2 Vertical asymptotes

An *asymptote* is a line to which a curve gets closer and closer as x approaches a certain value or as x goes to infinity or negative infinity.

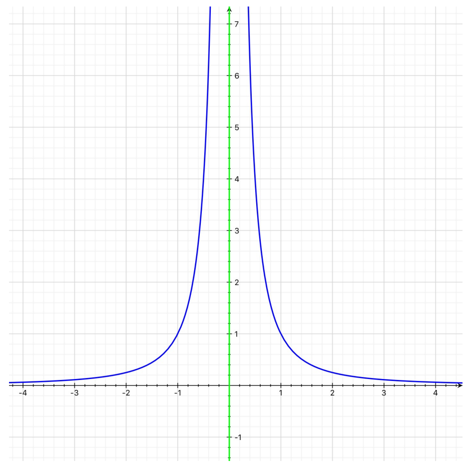
Consider the function $R(x) = 1/x$.



$y = 1/x$

Because the denominator contains a variable, it can equal zero. Where that happens, the function is undefined. Near that point, the y-value becomes indefinitely large (in absolute value). The line $x = 0$ which the curve approaches but never reaches, is a vertical *asymptote.* Values to the left of this asymptote are negative and those to the right are positive. That is, the function changes sign as it passes $x = 0$. That is because x – which is the whole denominator – changes sign at that point.

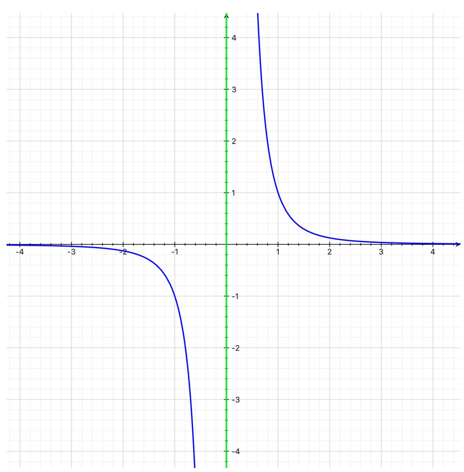
Now consider the function $R(x) = 1/x^2$.



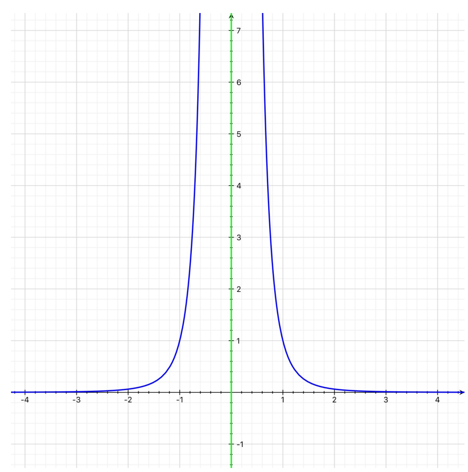
$y = 1/x^2$

The most striking difference between the graph of $1/x^2$ and that of $1/x$ is that for $1/x^2$ all y-values are positive or zero. That is because x^2, unlike x, is always positive or zero, so as the function passes $x = 0$ there is no change in sign – only a vertical asymptote.

What happens to the graph of a rational function as the power of x in the denominator goes up?



$y = 1/x^3$



$y = 1/x^4$

The function's sign changes at the asymptote if the power of x there is odd, but not if it's even. That's because the sign of x^n changes at the asymptote if n is odd, but not if it's even. You may want to play with this on a graphing calculator to verify that it works no matter how high n gets, as long as n is a positive integer.

3 Horizontal and slant asymptotes

3.1 Horizontal asymptotes

Some rational functions approach horizontal asymptotes as x gets more extreme, positive or negative. Which functions do that, and at what y-values do the asymptotes occur?

3.1.1 Case 1: The degree of the numerator is less than the degree of the denominator.

As you can see from the graphs above, the graph of 1 divided by x to a power goes to zero as x goes to extremes. (It is assumed that the power is a positive integer.) If you multiply the numerator by any constant (a number that does not contain a variable), you'll see that the graph of any number divided by x to a power goes to zero as x goes to extremes. **A rational function in which the degree of the numerator is less than the degree of the denominator has a horizontal asymptote in which y=0.**

3.1.2 Case 2: The degree of the numerator equals the degree of the denominator.

For this case, consider a complicated rational function like $R(x) = \frac{2x^4+4x^2+2}{3x^4+4x^3+1}$. To get a picture of what happens to this function as x goes to extremes, multiply both numerator and denominator by $\frac{1}{x^4}$:

$$R(x) = \frac{2x^4 \cdot \frac{1}{x^4} + 4x^2 \cdot \frac{1}{x^4} + 2 \cdot \frac{1}{x^4}}{3x^4 \cdot \frac{1}{x^4} + 4x^3 \cdot \frac{1}{x^4} + 1 \cdot \frac{1}{x^4}}$$

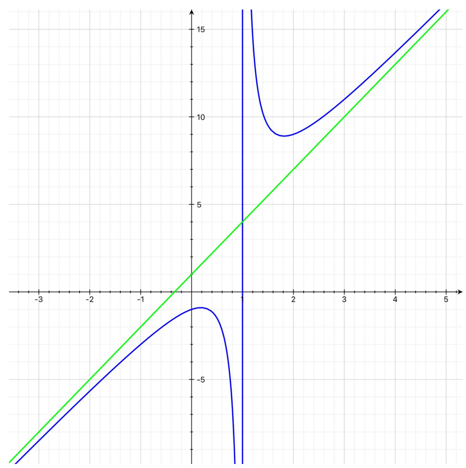
Carry out all those multiplications and you get:

$$R(x) = \frac{2 + \frac{4}{x^2} + \frac{2}{x^4}}{3 + \frac{4}{x} + \frac{1}{x^4}}$$

Except for the 2 in the numerator and the 3 in the denominator, all the terms in the numerator and denominator are rational functions in which the degree of the numerator is less than that of the denominator – and, as established above, as x goes to extremes these terms all go to zero. Then the function itself is equal to what's left: $\frac{2}{3}$. **A rational function in which the degree of the numerator equals the degree of the denominator has a horizontal asymptote in which y equals the quotient of the coefficients of the leading terms of numerator and denominator.** (It is assumed that the terms are arranged in order of decreasing degree.)

3.2 Slant asymptotes: The degree of the numerator is one more than the degree of the denominator.

Consider the rational function $y = \frac{3x^2-2x+1}{x-1}$. You can treat this function as the division problem that it is. Carry out the division and you get $\frac{3x^2-2x+1}{x-1} = 3x+1, r2$ or $\frac{3x^2-2x+1}{x-1} = 3x+1+\frac{2}{x-1}$. What happens to this function as x changes? As x becomes extreme in either direction, positive or negative, $\frac{2}{x-1}$ becomes smaller and the function's value gets closer to the line $3x+1$. $y = 3x+1$ a slant asymptote for this function.



$y = \frac{3x^2-2x+1}{x-1}$

Just as $\frac{2}{x-1}$ becomes smaller at extremes of x, so it becomes large in absolute value near the x-value where the denominator equals zero, at the vertical asymptote. The function's y-value is farthest from the slant asymptote where the x-value is near the vertical asymptote.

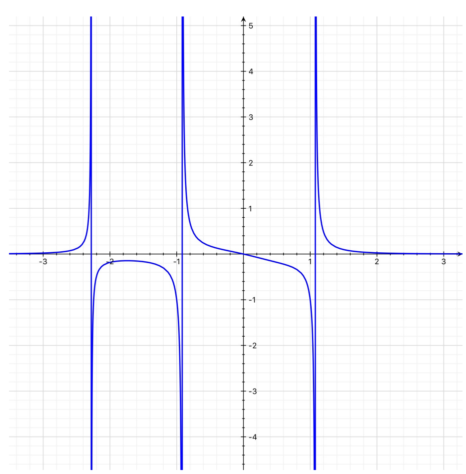
A slant asymptote occurs in a rational expression where the degree of the numerator is one more than the degree of the denominator. To find the slant asymptote, treat the rational function like a division problem and carry out the division. The slant asymptote is the quotient, not including the remainder.

4 Zeros

The x-value of an x-intercept is called a *zero*. In other words, **a zero is an x-value at which y=0.**

Consider the function $R(x) = \frac{x}{2x^5+3x^4-4x^3+x-3}$. Where does that function have a zero?

The graph crosses the x-axis where its y-value is zero. No matter how messy the denomimator, the y-value of a rational expression in simplest form is zero when and only when the expression's numerator is equal to zero. That means that as long as there are no common factors between the numerator and the denominator, you need not worry about the denominator when you are looking for zeros; just set the numerator equal to zero. For this example, that means there is a zero where $x = 0$.



$$y = \frac{x}{2x^5+3x^4-4x^3+x-3}$$

The numerator of any rational expression is a polynomial. According to the factor theorem, if a polynomial has a rational factor $x - c$, then it has a zero at $x = c$.

Consider the rational function $R(x) = \frac{(x+1)(x-2)(x-3)^2}{(x-1)(x-2)(x-4)^2}$.

To find this function's zeros, put it in simplest form; that is, cancel any factors that are common to the numerator and the denominator. For this function that's just $x - 2$.

$$R(x) = \frac{(x+1)(x-3)^2}{(x-1)(x-4)^2}$$

Regardless of what's left in the denominator, the function's zeros happen at x-values where the numerator equals zero:

$$\begin{array}{c|c} x+1=0 & x-3=0 \\ x=-1 & x=3 \end{array}$$

The function $R(x) = \frac{(x+1)(x-2)(x-3)^2}{(x-1)(x-2)(x-4)^2}$ has zeros at $x = -1$ and $x = 3$.

4.1 Touching and crossing the x-axis

At a zero – that is, a place where $y = 0$ – the graph may either cross the $x-$ axis or just touch it, kind of bounce off of it.

Why touch at some zeros and cross at others? Consider again the simplified rational function $R(x) = \frac{(x+1)(x-3)^2}{(x-1)(x-4)^2}$, which, as shown above, as zeros at $x = -1$ and $x = 3$. Note that the factor $x + 1$ shows up just once in the numerator. As the value of x goes from less than -1 to greater than -1, $x + 1$ changes from negative to positive and the function's sign changes. Thus the graph crosses the $x-$axis at this point. On the other hand, the factor $x - 3$ shows up twice – it's squared. As the value of x goes from less than 3 to greater than 3, $(x-3)^2$ goes from positive to zero to positive. It touches the $x-$ axis at $x = 3$. **Generally, a rational function's graph *crosses* the x-axis at a zero that corresponds to a factor that has an *odd* multiplicity** – that is, that occurs an odd number of times in the numerator. **A rational function *touches* the x-axis at a zero that corresponds to a factor that has an *even* multiplicity** – that is, that occurs an even number of times in the numerator.

5 Transformations

Consider the rational function $R(x) = 1/(x - 2)$. That looks a lot like $R(x) = 1/(x)$ above, but for the 2 that gets subtracted from x in the denominator. What is the effect of that change? To explore, set up a table and compare values.

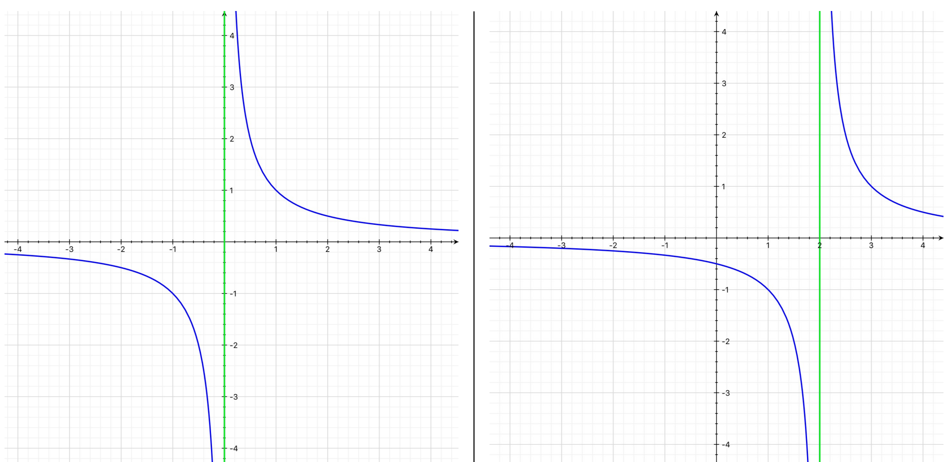
;

x	R(x)=1/x	x-2	S(x)=1/(x-2)
-3	$-1/3$	-5	$-1/5$
-2	$-1/2$	-4	$-1/4$
-1	-1	-3	$-1/3$
0	undefined	-2	$-1/2$
1	1	-1	-1
2	1/2	0	undefined
3	1/3	1	1

;

;

Note that $R(x) = 1/x$ and $S(x) = 1/(x-2)$ take on the same y-values, in the same sequence – but $S(x)$ takes those values 2 units to the right of where $R(x)$ takes them. Thus the curve for $S(x)$, which contains the term $x - 2$ in its denominator, falls 2 units to the right of the curve for $R(x)$.



$R(x) = \frac{1}{x}$ $\qquad$ $S(x) = \frac{1}{x-2}$

Students often find it counterintuitive that subtracting a number from x causes the graph to move to the right. Since you're subtracting, shouldn't it move to the left? There are a couple of ways to think about this.

At any value of x the graph of $y = \frac{1}{x-2}$ is doing at what the graph of $y = \frac{1}{x}$ is doing 2 units back. So at, for example, $x = 3$, the graph of $y = \frac{1}{x-2}$ is doing what the graph of $y = \frac{1}{x}$ is doing at $x = 3 - 2 = 1$. That has the effect of pushing the graph of $y = \frac{1}{x-2}$ *forward* 2 units.

Another way to look at it: For every value of x, $y = \frac{1}{x} + 2$ is 2 units higher than $y = \frac{1}{x}$ Why? Because you just added 2 to the *opposite* side of the equation. Shouldn't it work the same way for x? Let's solve for x in both $y = x$ and $y = 1/x$ and see what that looks like:

$$\begin{array}{r|l} y = \frac{1}{x} & y = \frac{1}{x-2} \\ x = \frac{1}{y} & x - 2 = \frac{1}{y} \\ & x = \frac{1}{y} + 2 \end{array}$$

So subtracting 2 from x does indeed have the effect of adding 2 to each x-value.

6 Sign

As you'll see below, sign is an important clue in graphing rational functions. Let's consider sign and how it changes for one rational function, $R(x) = \frac{(x+1)(x-2)(x-3)^2}{(x-1)(x-2)(x-4)^2}$, which we looked at in section 4, Zeros.

Note that in a rational function, sign changes only at zeros and vertical asymptotes, and not even all of those.

In simplest form, $R(x) = \frac{(x+1)(x-3)^2}{(x-1)(x-4)^2}$.

6.0.1 Zeros

Sign changes where a function crosses the x-axis. That includes every zero in the numerator with an odd multiplicity. For this function that's just $x = -1$.

6.0.2 Vertical asymptotes

The simplified function $R(x) = \frac{(x+1)(x-3)^2}{(x-1)(x-4)^2}$ has vertical asymptotes at the two x-values where the denominator equals zero: $x = 1$ and $x = 4$. But sign changes only at $x = 1$, corresponding to the factor $x - 1$, which is in the denominator an odd number of times (just once). Because the factor $x - 4$ occurs to the second power in the denominator, the function's sign does not change at $x = 4$.

6.0.3 Putting it together

Below are the places where the function changes sign and the intervals where sign is constant.

x-value	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$	$x = 3$	$3 < x < 4$	$x = 4$	$x > 4$
y-value		0		undefined		0		undefined	

Because the function does not change sign anywhere else, once you find the function's sign between the places where it does change – zeros and vertical asymptotes – you know the sign everywhere in the function's domain.

What sign does the function take in each interval?

To find the sign within an interval, you can evaluate the function at any point in the interval or you can evaluate the sign of each factor in the numerator and each factor in the denominator. If the number of negative factors is odd, the function is negative; if the number of negative factors is even, the function is positive.

Consider the left-most interval, where $x < -1$. The terms $x - 1$ (in the numerator) and $x + 1$ (in the denominator) are both negative in this interval. The other two terms, $(x - 3)^2$ and $(x - 4)^2$, are positive, because square terms can only be positive or zero, and these are not zero here. In fact, these terms are never negative, so except where they equal zero we can ignore their effect on the function's sign. In the left-most interval, the function is a negative number divided by a negative number, and the result is positive.

x-value	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$	$x = 3$	$3 < x < 4$	$x = 4$	$x > 4$
y-value	$+$	0		undefined		0		undefined	

Now consider the next interval to the right, $-1 < x < 1$. In the last interval, to the left of zero, $x + 1$ was negative. Now it has become positive. That leaves one negative, $x - 1$, in the function, making it negative.

x-value	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$	$x = 3$	$3 < x < 4$	$x = 4$	$x > 4$
y-value	$+$	0	$-$	undefined		0		undefined	

At $x = 1$, the function is undefined because its denominator equals zero.

To the right of $x = 1$, no terms change sign in either the function's numerator or its denominator, so the function does not change sign to the right of $x = 1$.. Except where its value is zero (at $x = 3$) or is undefined (at $x = 4$,) a vertical asymptote, the function is positive everywhere to the right of $x = 1$.

x-value	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$	$x = 3$	$3 < x < 4$	$x = 4$	$x > 4$
y-value	$+$	0	$-$	undefined	$+$	0	$+$	undefined	$+$

7 Graphing

Let's continue with the same example from earlier sections, $R(x) = \frac{(x+1)(x-2)(x-3)^2}{(x-1)(x-2)(x-4)^2}$.

These are the things you can look at to get the picture:

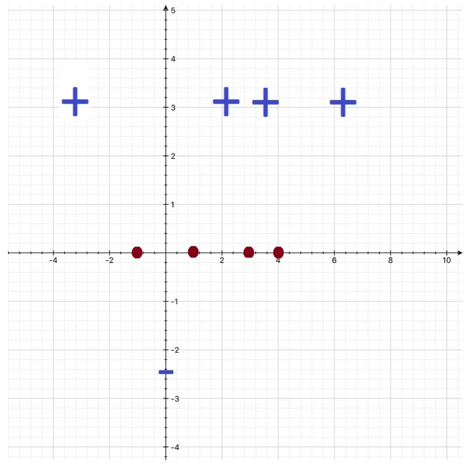
- sign as it changes through the intervals;
- vertical asymptotes;
- end behavior, including horizontal and slant asymptotes, or what happens when there is no horizontal or slant asymptote;
- y-intercept;
- x-intercepts;
- holes;
- some other points;

We'll work with the simplified form of the function but keep in mind that it is the simplified form – the part that got lost in the simplification will create a hole.

As shown above, the simplified form of the function is $R(x) = \frac{(x+1)(x-3)^2}{(x-1)(x-4)^2}$.

7.1 Sign

Start the graph by indicating the signs that the function takes across its domain.

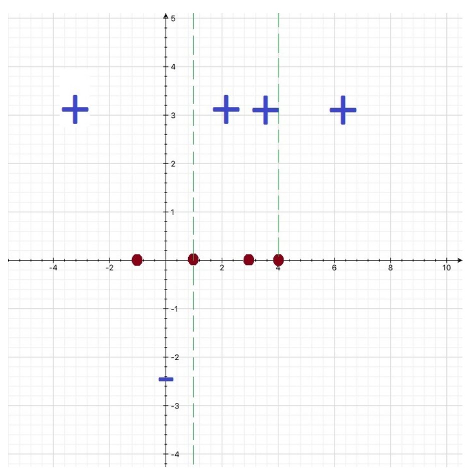


7.2 Vertical asymptotes

There are vertical asymptotes where the denominator of the simplified expression equals zero.

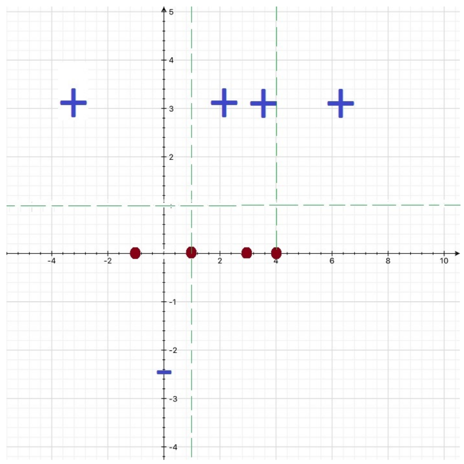
$$R(x) = \frac{(x+1)(x-3)^2}{(x-1)(x-4)^2}$$

There are vertical asymptotes at x=1 and at x=4. The asymptote at $x = 1$ comes from a factor whose multiplicity, 1, is odd, so the graph goes in opposite directions on opposite sides of the asymptote (to negative infinity on the left and positive infinity on the right), per the sign diagram in section 6; the asymptote at $x = 3$ comes from a factor whose multiplicity, 2, is even, so it goes in the same direction (to positive infinity) on both sides.



7.3 End behavior

To explore end behavior, compare the degree of the numerator with that of the denominator. In the rational function $R(x) = \frac{(x+1)(x-3)^2}{(x-1)(x-4)^2}$, both numerator and denominator have degree 3. When the degrees of numerator and denominator are equal, there is a horizontal asymptote in which y is equal to the quotient of the coefficients of numerator and denominator. In this case, both numerator and denominator have an implied quotient of 1, so **the horizontal asymptote is y=1**.

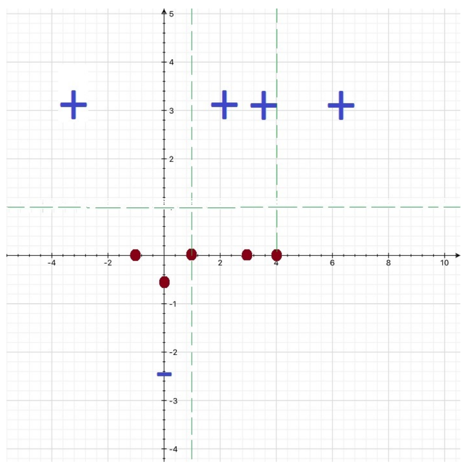


7.4 y-intercept

The y-intercept is the point at which $x = 0$. To find it, substitute 0 for x in the function's equation.

$$R(x) = \frac{(x+1)(x-3)^2}{(x-1)(x-4)^2}$$
$$R(0) = \frac{(0+1)(0-3)^2}{(0-1)(0-4)^2}$$
$$R(0) = \frac{(1)(-3)^2}{(-1)(-4)^2}$$
$$R(0) = \frac{9}{-16} = -\frac{9}{16}$$

The y-intercept is at (0, -9/16).



7.5 x-intercepts

See the section on "zeros," above. An x-intercept is just another name for a zero. Because we're given this function in factored form, the zeros are easy to find: Each x-value at which a factor equals zero is a zero. **The zeros are x=-1 and x=3.**

7.6 Holes

A function has a hole at an x-value where the denominator is zero in the function's original, unsimplified form but not in the simplified form. At a hole, the canceled term makes the function undefined; everywhere else, the simplified function is the same as the original function.

In the original function, $R(x) = \frac{(x+1)(x-2)(x-3)^2}{(x-1)(x-2)(x-4)^2}$, the term $x-2$ shows up in both the numerator and the denominator. $x-2=0$ when $x=2$, so there is a hole at $x=2$.

What is the hole's y-value? In the simplified form of the equation, $R(x) = \frac{(x+1)(x-3)^2}{(x-1)(x-4)^2}$, $R(2) = \frac{(2+1)(2-3)^2}{(2-1)(2-4)^2} = \frac{3}{4}$.

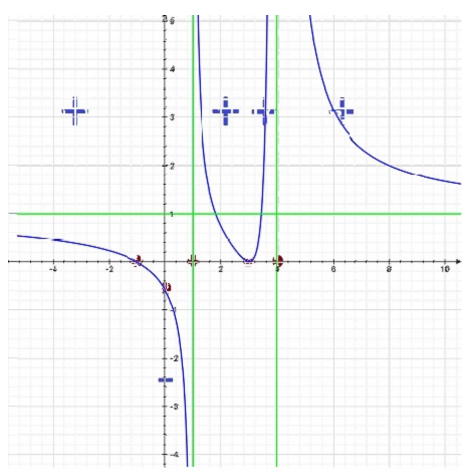
The hole is at (2, 3/4).

7.7 Points

How many points do you need to sketch a graph? Different instructors have different requirements, and it really depends on how exact your graph needs to be. My personal advice would be to shoot for one representative point in each interval.

7.7.1 Sketching

Time to put together all the information you've gathered. Start by graphing the asymptotes. That will give you a framework. Add the points you've got.



$$y = \frac{(x+1)(x-2)(x-3)^2}{(x-1)(x-2)(x-4)^2}$$

8 Solving Rational Equations

Consider the rational equation $\frac{3}{x+1} = \frac{2}{x-1} + \frac{2}{(x+1)(x-1)}$. How would you go about solving it?

What makes this equation complicated is the rational expression format – in other words, it has denominators. Multiply both sides by the least common multiple of the denominators to make them go away. That's called clearing the fractions.

$$\begin{gathered} \frac{3}{x+1} = \frac{2}{x-1} + \frac{2}{(x+1)(x-1)} \\ 3(x-1) = 2(x+1) + 2 \\ 3x - 3 = 2x + 2 + 2 \\ x = 7 \end{gathered}$$

When you clear the fractions, you lose information. A rational expression is undefined anyplace its denominator is zero. Multiplying by the denominators does not change the domain, but it hides the domain constraint. Because domain information is now hidden, you need to check answers to make sure they do not take you into undefined oblivion. What does that look like? Consider this example:

$$\frac{3x^2}{x+1} = \frac{2x^2-4x+2}{x-1} + \frac{3}{x+1}$$

Multiply by the least common multiple of the denominators:

$$\begin{gathered} (x+1)(x-1) \cdot \frac{3x^2}{x+1} = (x+1)(x-1) \cdot \frac{2x^2-4x+2}{x-1} + (x+1)(x-1) \cdot \frac{3}{x+1} \\ (x-1) \cdot (3x^2) = (x+1) \cdot (2x^2 - 4x + 2) + (x-1) \cdot 3 \\ 3x^3 - 3x^2 = 2x^3 - 2x^2 - 2x + 2 + x - 1 \\ x^3 - x^2 - x + 1 = 0 \\ x^2(x-1) - 1(x-1) = 0 \\ (x^2-1)(x-1) = 0 \\ x = -1, 1 \end{gathered}$$

Now check those answers in the original equation, $\frac{3x^2}{x+1} = \frac{2x^2-4x+2}{x-1} + \frac{3}{x+1}$:

For $x = -1$	For $x = 1$
$\frac{3(-1)^2}{-1+1} = \frac{2(-1)^2-4(-1)+2}{-1-1} + \frac{3}{-1+1}$	$\frac{3 \cdot 1^2}{1+1} = \frac{2 \cdot 1^2 - 4 \cdot 1 + 2}{1-1} + \frac{3}{1+1}$

In each case there is a denominator equal to zero. Both roots are extraneous and the equation has no solution.

9 Solving Rational Inequalities

Consider an inequality related to one of the rational equations we've already looked at: $\frac{3}{x+1} > \frac{2}{x-1} + \frac{2}{(x+1)(x-1)}$. For the equation, the first step was to multiply by the denominators. For the inequality, you can't that; in an inequality, when you multiply by a negative number, you switch the direction of the inequality sign – but the denominators change sign, so changing the direction of the inequality side is tricky. You need a different strategy.

It is easier to compare a quantity to zero than to compare it to a different number. Subtract from both sides of the equation everything that is on the left-hand side, and you will be comparing a rational expression to zero:

$$\frac{3}{x+1} > \frac{2}{x-1} + \frac{2}{(x+1)(x-1)}$$
$$0 > \frac{2}{x-1} + \frac{2}{(x+1)(x-1)} - \frac{3}{x+1}$$
$$\frac{2}{x-1} + \frac{2}{(x+1)(x-1)} - \frac{3}{x+1} < 0$$

That last step is just to get to the more familiar format of having the zero on the right and everything else on the left.

To compare the left-hand side of the inequality to zero is to determine whether it is positive, negative, or zero. To do that, find the places where the expression is zero or changes sign. Between those places, the expression will be either consistently positive or consistently negative.

Where can sign changes happen? At asymptotes and zeros.

The function $\frac{2}{x-1} + \frac{2}{(x+1)(x-1)} - \frac{3}{x+1}$ has asymptotes where at least one of its denominators equals zero: at $x = -1$ and $x = 1$. (This approach to finding asymptotes works only for expressions in simplest form.) Let's put that into a table:

x-value	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 7$	$x = 7$	$x > 7$
y-value		undefined		undefined			

For what value or values of x is $\frac{2}{x-1} + \frac{2}{(x+1)(x-1)} - \frac{3}{x+1}$ equal to zero? To find it or them, rewrite the expression with a common denominator and set it equal to zero:

$$\frac{2}{x-1} + \frac{2}{(x+1)(x-1)} - \frac{3}{x+1} = 0$$
$$\frac{2\cdot(x+1)+2-3(x-1)}{(x-1)(x+1)} = 0$$

The expression equals zero when its numerator equals zero:

$$2x + 2 + 2 - 3x + 3 = 0$$
$$x = 7$$

So there is a zero at $x = 7$.

x-value	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 7$	$x = 7$	$x > 7$
y-value		undefined		undefined		0	

Now find the sign in the remaining intervals. To find the sign where $x < -1$, substitute into the expression what you think is likely the easiest number less than -1 for which to calculate the function. I think the easiest number will be -2.

$$\frac{2}{x-1} + \frac{2}{(x-1)(x+1)} - \frac{3}{x+1}$$
$$\frac{2}{-2-1} + \frac{2}{(-2-1)(x+1)} - \frac{3}{-2+1}$$
$$-\frac{2}{3} + \frac{2}{3} + 3 = 3$$

That's a positive result, so the expression is positive for x values less than 1:

x-value	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 7$	$x = 7$	$x > 7$
y-value	$+$	undefined		undefined		0	

Next consider the interval between -1 and 1. Zero looks like a convenient number:

$$\frac{2}{x-1} + \frac{2}{(x-1)(x+1)} - \frac{3}{x+1}$$
$$\frac{2}{0-1} + \frac{2}{(0-1)(0+1)} - \frac{3}{0+1}$$
$$-2 - 2 - 3 = -7$$

That's a negative result, so the expression is negative for x values between -1 and 1:

x-value	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 7$	$x = 7$	$x > 7$
y-value	$+$	undefined	$-$	undefined		0	

Next consider the interval between $x = 1$ and $x = 7$. Try $x = 2$:

$$\frac{2}{x-1} + \frac{2}{(x-1)(x+1)} - \frac{3}{x+1}$$
$$\frac{2}{2-1} + \frac{2}{(2-1)(2+1)} - \frac{3}{2+1}$$
$$2 + \frac{2}{3} - 1 = \frac{5}{3}$$

That's a positive result, so the expression is positive for x values between 1 and 7:

x-value	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 7$	$x = 7$	$x > 7$
y-value	$+$	undefined	$-$	undefined	$+$	0	

Finally, consider numbers greater than 7. Try 8:

$$\frac{2}{x-1} + \frac{2}{(x-1)(x+1)} - \frac{3}{x+1}$$
$$\frac{2}{8-1} + \frac{2}{(8-1)(8+1)} - \frac{3}{8+1}$$
$$\frac{2}{7} + \frac{2}{63} - \frac{3}{9}$$
$$\frac{18}{63} + \frac{2}{63} - \frac{21}{63} = -\frac{1}{63}$$

That's a negative answer, so $\frac{2}{x-1} + \frac{2}{(x-1)(x+1)} - \frac{3}{x+1}$ is negative everywhere to the right of 7.

x-value	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 7$	$x = 7$	$x > 7$
y-value	$+$	undefined	$-$	undefined	$+$	0	$-$

The original problem was the solve the inequality $\frac{3}{x+1} > \frac{2}{x-1} + \frac{2}{(x+1)(x-1)}$, which, we found, is equivalent to $\frac{2}{x-1} + \frac{2}{(x+1)(x-1)} - \frac{3}{x+1} < 0$. The solution to this inequality is all values where the table shows a number less than zero: $(-1, 1) \cup (7, \infty)$.

Questions? Reach out to us at the lab: 703.450.2644 or LOMathCenter@nvcc.edu.