Practice Calculus Test without Trig

The problems here are similar to those on the practice test. Slight changes have been made.

1. What is the domain of the function $f(x) = \sqrt{3x-1}$? Express the answer in interval notation.

   The domain is the set of values x can take. Unless there is reason to believe otherwise, it is usually assumed that $f(x)$ is real. For $f(x)$ to be real, the expression under the radical sign must be 0 or greater. Express that as an inequality and solve:

   $$3x - 1 \geq 0$$
   $$3x \geq 1$$
   $$x \geq 1/3$$

   In interval notation, that's $[1/3,\ \infty)$.

2. What is the range of $f(x) = x^2 - 9$? Express the answer in interval notation.

   The range is the set of values y can take. What values can y take here? Note that x^2 is always positive or 0. As the absolute value of x gets large, $f(x)$ will increase without limit, so the x^2 part of $f(x)$ will be 0 at least and possibly much more. The value 9 will be subtracted from x^2. When $x = 0,\ f(x) = -9.$ When $x \neq 0,\quad f(x) > -9$, with no limit to how high it can go as the absolute value of x increases.

   In interval notation, that's $[-9,\ \infty)$.

3. Find the composition of the functions $f(x) = e^{2x}$ and $g(x) = 2x + 4,\ (f \circ g)(x)$.

   To find $(f \circ g)(x)$, perform the function $f(x)$ on $g(x)$, treating $g(x)$ as if it were x. In other words, $(f \circ g)(x) = f(g(x))$.

   $$(f \circ g)(x) = f(2x+4) = e^{2(2x+4)} = e^{(4x+8)}$$

4. Simplify $(2a^{-3})^4$.

Consider these rules for exponents:

Rule	Example	How It Applies Here
$(bc)^n = b^n c^n$	$(2 \cdot 3)^2 = (2 \cdot 3)(2 \cdot 3) = 2^2 \cdot 3^2$	$(2a^{-3})^4 = 2^4(a^{-3})^4$
$(b^c)^d = b^{cd}$	$\left(2^3\right)^2 = (2 \cdot 2 \cdot 2)^2 = 8^2 = 64 = 2^6$	$2^4(a^{-3})^4 = 16a^{-12}$
$b^{-c} = 1/b^c$	$2^{-1} = 1/2$	$16a^{-12} = 16/a^{12}$

The final answer is $16/a^{12}$.

5. Simplify $x^{2a+3}x^{4a}$.

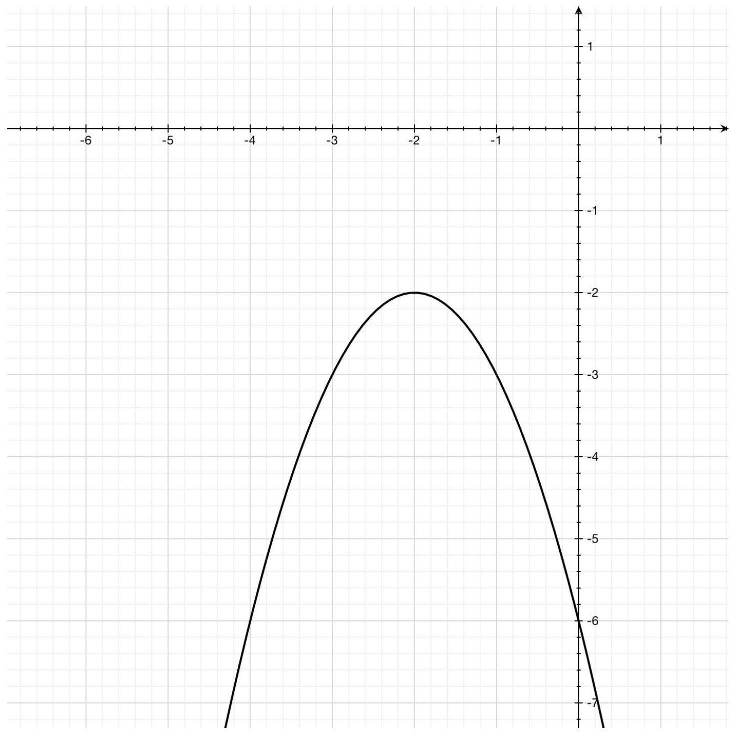
To multiply a base taken to a power by the same base taken to a power (either the same power or a different one), add the exponents. The product is the same base taken to the **sum** of the powers.

People want to multiply exponents here, but that's not right. Consider a simple example: $3^2 \cdot 3^4 = (3 \cdot 3)(3 \cdot 3 \cdot 3 \cdot 3)$ Count up all the 3's and you see that that's 3^6. And $6 = 2 + 4$, the sum of the exponents.

It works the same with complicated algebraic expressions: Add the exponents. In the problem $x^{2a+3}x^{4a}$,

$2a + 3 + 4a = 6a + 3$, so
$x^{2a+3}x^{4a} = x^{6a+3}$.

6. Write the equation of the parabola shown in the graph.



The equation of a parabola in vertex format, the format most useful for this problem, looks like $f(x) = a(x-h)^2 + k$, where (h, k) is the location of the vertex.

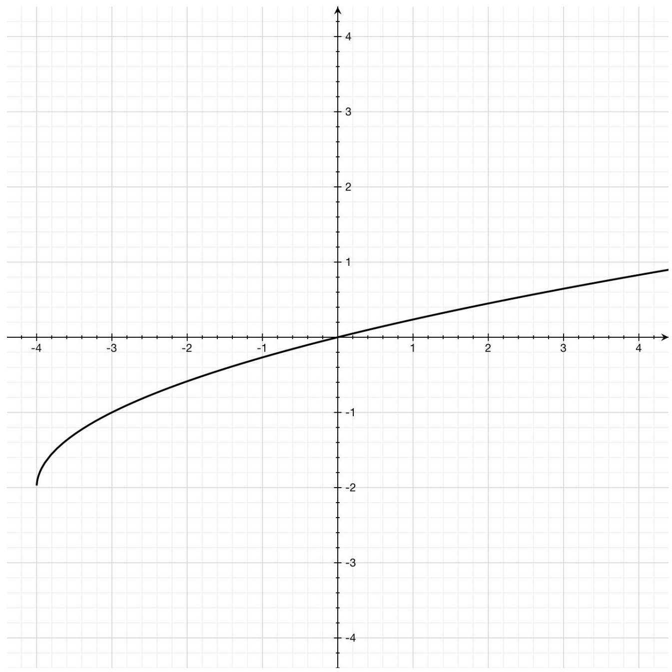
This parabola's vertex, as you can see, is at $(-2,\ -2)$. So its equation is $f(x) = a(x+2)^2 - 2$. You still have to find a. To do that, substitute the coordinates of another point on the parabola into the equation you have. Any point on the parabola will do, but usually a point with whole-number coordinates (so you won't have to deal with fractions) and, if you're lucky, also with an $x-$value one unit away from that of the vertex, is easiest. Try $(-1,\ -3)$, a point you can see on the graph. Substitute -3 for $f(x)$ and -1 for x.

$$-3 = a(-1+2)^2 - 2$$
$$-1 = a(1)^2$$

$-1 = a$ or $a = -1$

Then $f(x) = -(x+2)^2 - 2$ and that's the parabola's equation.

7. Write the equation of the square root function shown in the graph.



The equation you need will be in the form $f(x) = a\sqrt{x-h} + k,$ where $(h,\ k)$ are the coordinates of the vertex of what is, after all, the upper half of a rotated parabola. On this graph that is the left-hand endpoint. You can read its coordinates: $(-4,\ -2)$. So the graph's equation is $f(x) = a\sqrt{x+4} - 2.$ To find $a,$ substitute the coordinates of another point on the graph into the equation. The origin $(0,\ 0)$ looks like an easy point on the graph:

$$0 = a\sqrt{0+4} - 2$$
$$a\sqrt{4} = 2$$
$$2a = 2$$
$$a = 1$$

Finally, the equation is $f(x) = \sqrt{x+4} - 2.$

8. Given $f(x) = 7x^3 + 3$, find $f^{-1}(x).$

The term $f^{-1}(x)$ means the inverse of $f(x)$. The inverse undoes the original function. It is the function you get if you reverse the roles of x and y in the original function.

So do that: reverse the roles of x and y in the original function:
$y = 7x^3 + 3$
Reverse: $x = 7y^3 + 3$

But you need to show this relation with y isolated on the left-hand side, so solve for y :

$$x = 7y^3 + 3$$
$$x - 3 = 7y^3$$
$$\frac{x-3}{7} = y^3$$
$$y = \sqrt[3]{\frac{x-3}{7}}$$
$$f^{-1}(x) = \sqrt[3]{\frac{x-3}{7}}$$

And that's the correct answer.

9. Graph the function

$$f(x) = \begin{cases} 3^x \text{ for } x \le 1 \\ 4x - 1 \text{ for } x > 1 \end{cases}$$

Find the $y-$values for several $x-$values in each piece of this piecewise-defined function:

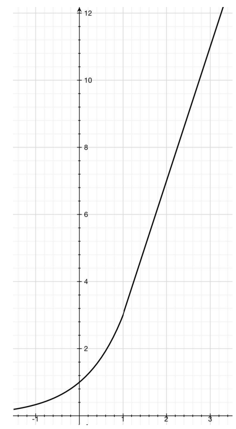
x	$f(x)$
-1	1/3
0	1
1	3
Just a tiny bit >1	Just a tiny bit >3
2	7
3	11

The part on the left, for $x \le 1,$ is an exponential equation with a base that is greater than 1. The graph of such an equation is asymptotic to the $x-$axis as x

approaches negative infinity. As you go further to the right, the graph increases at an increasing rate. At the end of the left-hand piece, where $x = 1,\ 3^x = 3.$

The other piece, which picks up just after $x = 1,$ is a straight line with a positive slope of $4.$ If this piece did extend left all the way to $x = 1$ -- instead of ending just to the left of $x = 1$ -- the two pieces would meet at the point $(1, 3)$.

It graphs up like this:



10. Given

$$f(x) = \begin{cases} log\,(x-2) & \text{for } x \leq 4 \\ x^3 - 2 & \text{for } x > 4 \end{cases}$$

evaluate $f(3)$.

Note that only one piece of this function is relevant to the problem. You're asked to evaluate the function for an $x-$value less than $4,$ so consider only the piece for $x \leq 4$: $f(x) = log\,(x-2)$.

$y = log\,(3-2)$
$y = log\,(1)$
$y = 0$

11. Given $f(x) = 5x + 2,$ simplify
$\frac{f(x+h)-f(x)}{h}$
If $f(x) = 5x + 2,$ then $f(x+h) = 5\,(x+h) + 2.$ Then
$\frac{f(x+h)-f(x)}{h} = \frac{5(x+h)+2-(5x+2)}{h} = \frac{5x+5h+2-5x-2}{h} = \frac{5h}{h} = 5$
The answer is $5.$

12. Given $f(x) = 3x^2 - 4x$, simplify $\frac{f(x+h)-f(x)}{h}$.

Note that:

$f(x+h) = 3(x+h)^2 - 4(x+h)$

$= 3\left(x^2 + 2hx + h^2\right) - 4x - 4h$

$= 3x^2 + 6hx + 3h^2 - 4x - 4h$

Then

$\frac{f(x+h)-f(x)}{h} = \frac{3x^2+6hx+3h^2-4x-4h-(3x^2-4x)}{h}$

$= \frac{6hx+3h^2-4h}{h}$

$= 6x + 3h - 4$

13. Rationalize the denominator.

$\frac{2}{\sqrt{x}-4}$

To rationalize the denominator, you need to multiply the expression by 1 in the form of the denominator's conjugate divided by itself.

$\frac{2}{\sqrt{x}-4} \cdot \frac{\sqrt{x}+4}{\sqrt{x}+4} = \frac{2\sqrt{x}+8}{x-16}$

14. Divide. Give your answer in $a + bi$ format.

$\frac{2}{4-3i}$

$\frac{2}{4-3i}$	
$\frac{2}{4-3i}\left(\frac{4+3i}{4+3i}\right)$	Multiply by 1 in the form of the denominator's complex conjugate divided by itself.
$= \frac{8+6i}{25}$	Carry out the multiplication.
$= \frac{8}{25} + \frac{6i}{25}$	Reformat into two terms.

15. Write as a single logarithm:
$log\,(4x+1) - log\,(x-2) - 4logx$

$4logx = logx^4$ Power rule for logarithms.

$log\,(4x+1) - log\,(x-2) - 4logx$ Apply the power rule.
$= log\,(4x+1) - log\,(x-2) - logx^4$

$= log\frac{(4x+1)}{(x-2)\cdot x^4}$ Quotient rule for logarithms.

16. Expand the expression $ln\left(\frac{xy^2}{4z}\right)$.

$ln\,(xy^2) = lnx + ln\,(y^2)$
$ln\,(4z) = ln4 + lnz$

Product rule for logarithms

$ln\,(y^2) = 2lny$

$ln\,(xy^2) = lnx + 2lny$

Power rule for logarithms

$ln\left(\frac{xy^2}{4z}\right) = ln\,(xy^2) - ln\,(4z)$

Quotient rule for logarithms

$ln\left(\frac{xy^2}{4z}\right) = lnx + 2lny - (ln4 + lnz)$
$ln\left(\frac{xy^2}{4z}\right) = lnx + 2lny - ln4 - lnz$

Substitution. Notice that the two terms in the denominator are both treated the same way. (It is a common error to add the second denominator term rather than subtract it.)

17. Write $\sqrt[5]{x^4}$ without the radical sign.

To write a radical without a radical sign, use exponential notation: $\sqrt[n]{y} = y^{\frac{1}{n}}$.
Apply that to $\sqrt[5]{x^4}$ and you get $\sqrt[5]{x^4} = x^{\frac{4}{5}}$.

18. Solve the following equation for w.

$v = log_5 (3w - 2)$

$3w - 2 = 5^v$	Basic logarithmic relationship: $y = log_a x$ is equivalent to $x = a^y$.
$w = \frac{5^v + 2}{3}$	Solve for w.

19. Solve the equation for x :

$log_3 x + log_3 (x - 8) = 2$

$log_3 [x(x - 8)] = 2$	Product rule for logarithms.
$log_3 (x^2 - 8x) = 2$	
$3^2 = x^2 - 8x$	Basic logarithmic relationship: $log_a b = c$ is equivalent to $a^c = b$.
$x^2 - 8x - 9 = 0$	Rewrite the equation in quadratic format.
$(x - 9)(x + 1) = 0$	Factor.
$x - 9 = 0$ $\quad x + 1$ $x = 9$ $\quad x = -1$	Solve.

Checking your work is always a good idea, but in logarithm problems it's especially important, because the domain of the logarithmic function is limited. If one of your answers falls outside the domain, that answer is *extraneous*, not a valid answer.

$$log_3 x + log_3 (x - 8) = 2$$

Check the answer $x = 9$:

$$log_3 9 + log_3 (9 - 8) =$$

$$log_3 9 + log_3 1 =$$

$2 + 0 = 2.$ Check.

Check the answer $x = -1$:

$$log_3 (-1) + log_3 (-1 - 8) =$$

$$log_3 (-1) + log_3 (-9) =$$

But what is $log_3 (-1)$ or $log_3 (-9)$? The domain of the log function includes only **positive** real numbers; an answer that requires you to take the log of a negative number is extraneous.