Domain

What Is Domain?

A function's *domain* is the set of values that x can take for that function.

When Is Domain Limited?

For this discussion's purposes, we assume all values of x and y are real. So in a function f, x can be any real number that corresponds to a real value of y for that function.

For many functions, including all polynomial functions -- that is functions of the form $a_1x^n + a_2x^{n-1} + a_3x^{n-2} + \ldots + a_{n+1}x^n$ -- the domain is all real numbers.

Let's consider three common circumstances in which a function's domain is limited:

- **Square roots or fourth roots or sixth roots** or any even roots of negative numbers. The square root of a negative number is imaginary. The fourth, sixth, and other even roots of a negative number contain imaginary parts. The x −values that cause the radicand (the number under the radical sign) of an even root to be negative are excluded from a function's domain. For example, for $f(x) = \sqrt{x-2}$, 0 is not included in the function's domain because if $x = 0$ then $y = \sqrt{-2}$, an imaginary number.
- Values of x that would make a function's **denominator equal zero**.
- **Real-world limits**. Say you're creating a function to find the amount of money you've earned since you started your job. That function can't start before your start date at your job, and can't continue past the day you leave.

How Can I Find a Function's Domain?

To find a function's domain, look for even roots, denominators that may take a value of zero, and limits that make sense in a real-world situation.

Examples

1. What is the domain of the function $g(x) = \sqrt[4]{x^2 + x}$?

Solution Note that the radical's index is an even number. That means the radicand must be greater than or equal to zero.

$$x^2 + x \geq 0$$
$$x(x+1) \geq 0$$

Then x is in this interval: $(-\infty, -1] \cup [0, \infty)$. That's where $x^2 + x \geq 0$ and therefore where $\sqrt[4]{x^2 + x}$ is real.

2. What is the domain of the function $h(x) = \frac{x+1}{x^2+3x+2}$?

Solution Values of x that would make a function's denominator equal zero are excluded from the domain. Let's see what values that might be.

$$x^2 + 3x + 2 = 0$$
$$(x + 2)(x + 1) = 0$$
$$x = -1, -2$$

x −values of -1 and -2 would cause the denominator to equal zero, so they are excluded from the domain. Then the domain is $(-\infty, -1) \cup (-1, -2) \cup (-2, \infty)$.

Note that the domain does not depend on the numerator, even in a function like $h(x)$, in which the numerator is a factor of the denominator.

3. A swimming pool's depth varies from one end of the pool to the other. The pool's length is l. Function $j(x)$ gives the pool's depth as a function of x, the distance from the pool's south end. What is the domain of function $j(x)$?

Solution The pool's depth has meaning only in the pool -- so the domain goes from $x = 0$ at the pool's south end to $x = l$ at the pool's north end, and the domain of $j(x)$ is $[0, l]$.

4. Question What is the domain of the function $k(x) = 5x^4 + (1/2)x^3 + x^2 - x + 1$?

Solution This is a polynomial function. As stated above, the domain of all polynomial functions is all real numbers.