# Practice Test 2

**1.** Find the domain and intercepts of the function $f(x) = \frac{x^3-4x+3}{2x^2-8}$.

**Answer:** $Domain: D_f = (-\infty, -2) \cup (-2,2) \cup (2, \infty); x - intercept: (1,0), (3,0); y - intercept: (0, -\frac{3}{8})$

**2.** Find the domain of the rational function $f(x) = \frac{x-2}{x^2+3x-10}$

**Answer:** $D = \{x | x \neq 2, x \neq -5\}$

**3.** Find all asymptotes of $f(x) = \frac{x-2}{x^2+3x-10}$

**Answer:** $x = -5$ and $y = 0$

**4.** Determine the end behavior of $f(x) = -3x^4 - 3x^2 + 5x - 7$

**Answer:** $f(x) \to -\infty$ as $|x| \to \infty$

**5.** Find the slant/oblique asymptote of $f(x) = \frac{3x^4-2x^3+x+1}{x^3+x-2}$

**Answer:** $y = 3x - 2$

**6.** Find the $x$-intercepts of $f(x) = x - \frac{2x+6}{x+1}$

**Answer:** 3 and -2

**7.** Factor completely (over the set of real numbers) $3x^3 - 2x - 1$

**Answer:** $3x^3 - 2x - 1 = (x-1)(3x^2 + 3x + 1)$

**8.** Determine the behavior of $f(x) = \frac{x-2}{x^2+3x-10}$ as $x$ approaches $-5$ from the left.

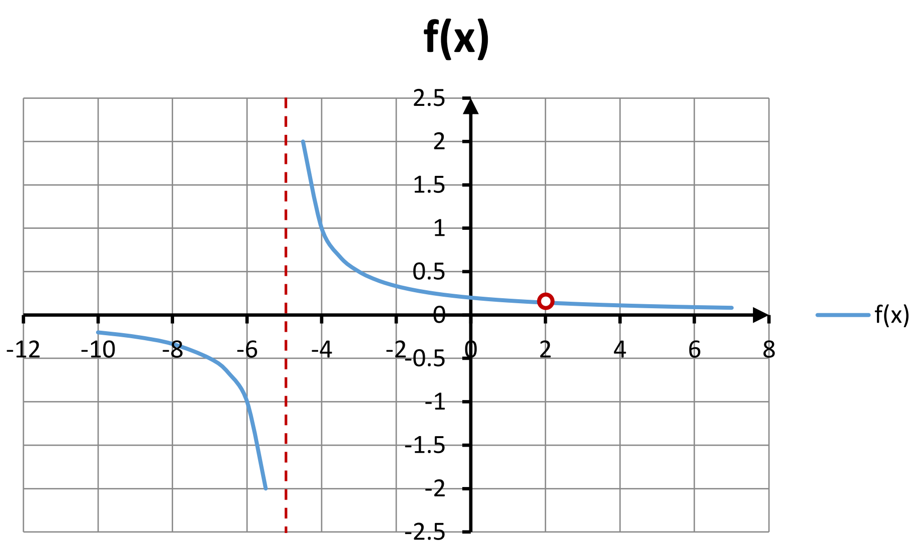
**Answer:** $f(x) \to -\infty$ as $x \to 5^-$

**9.** Solve $x - \frac{2x+6}{x+1} \geq 0$

**Answer:** $[-2,1) \cup [3, \infty)$

**10.** Graph $f(x) = \frac{x-2}{x^2+3x-10}$

**Answer:**




**11.** Determine the behavior of $f(x) = x^3 - 2x^2 + x$ near $x = 1$.

**Answer:** $f(x) \approx (x-1)^2$

**12.** Write the form of the partial fraction decomposition of the rational expression.

a. $\frac{3x-1}{x^2+x-6}$ **Answer:** $\frac{3x-1}{x^2+x-6} = \frac{A}{x-2} + \frac{B}{x+3}$

b. $\frac{x^2+11x+9}{(x+3)^2(x-2)}$ **Answer:** $\frac{x^2+11x+9}{(x+3)^2(x-2)} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x-2}$

c. $\frac{x^4+11x+9}{(x^2+x+3)^2(x-2)}$ **Answer:** $\frac{x^2+11x+9}{(x+3)^2(x-2)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+x+3} + \frac{Dx+E}{(x^2+x+3)^2}$

**13.** Find the decomposition of $\frac{x}{x^2-3x+2}$ into partial fractions using two ways.

**Answer:** $\frac{x}{x^2-3x+2} = \frac{2}{x-2} - \frac{1}{x-1}$

**14.** Expand $\frac{3x^2-x+4}{(x+1)(x^2-1)}$ into the sum of partial fractions.

**Answer:** $\frac{3x^2-x+4}{(x+1)(x^2-1)} = \frac{1}{x-1} - \frac{1}{x+1} + \frac{2}{(x+1)^2}$

**15.** Solve the system $\begin{cases} 3x - 2y = 7 \\ 5x + 2y = 1 \end{cases}$ using Gaussian elimination (by changing to row-echelon form)

**a.** Write the augmented matrix for the system:

$$\left[\begin{array}{cc|c} 3 & -2 & 7 \\ 5 & 2 & 1 \end{array}\right]$$

To get 1 in row 1, column 1, perform the row operation $2R_1 + (-1)R_2 \to R_1$:

$$\begin{array}{lcc|c} 2R_1: & 6 & -4 & 14 \\ (-1)R: & -5 & -2 & -1 \\ \hline new\ R_1: & 1 & -6 & 13 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & -6 & 13 \\ 5 & 2 & 1 \end{array}\right]$$

To get 0 in row 2, column 1, perform the row operation: $-5R_1 + R_2 \to R_2$.

$$\left[\begin{array}{cc|c} 1 & -6 & 13 \\ 0 & 32 & -64 \end{array}\right]$$

To get 1 in row 2, column 2 perform the row operation: $\frac{1}{32}R_2 \to R_2$.

$$\left[\begin{array}{cc|c} 1 & -6 & 13 \\ 0 & 1 & -2 \end{array}\right]$$

Write the system corresponding to the last augmented matrix and solve it using back substitution.

$$\begin{cases} x - 6y = 13 \\ y = -2 \end{cases}$$

$$x - 6(-2) = 13$$

$$x = 1$$

**16.** Solve the system $\begin{cases} 3x - 2y + z = 2 \\ x + y - z \quad = 0 \\ 2x + y - 2z = -2 \end{cases}$ using Gaussian Elimination.

**Solution**

Step 1: Write the augmented matrix of the system:

$$\begin{bmatrix} 3 & -2 & 1 & | & 2 \\ 1 & 1 & -1 & | & 0 \\ 2 & 1 & -2 & | & -2 \end{bmatrix}$$

Step 2: Get 1 in row 1, column 1.

In our case the easiest way is to interchange row 2 and row 1: $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & -1 & | & 0 \\ 3 & -2 & 1 & | & 2 \\ 2 & 1 & -2 & | & -2 \end{bmatrix}$$

Step 2: Perform row operations to get the entry 0's in the first column below the 1.
Perform the row operation $R_2 = -3r_1 + r_2$ to get 0 in row 2, column 1 and perform $R_3 = -2r_1 + r_3$ to get 0 in row 3, column 1.

$$\begin{bmatrix} 1 & 1 & -1 & | & 0 \\ 0 & -5 & 4 & | & 2 \\ 0 & -1 & 0 & | & -2 \end{bmatrix}$$

Step 4: Perform row operations to get 1 in row 2, column 2. Then get 0's below the 1.
Interchange row 2 and row 3, and multiply the new row 2 by $-1$:

$$\begin{bmatrix} 1 & 1 & -1 & | & 0 \\ 0 & -5 & 4 & | & 2 \\ 0 & -1 & 0 & | & -2 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 1 & -1 & | & 0 \\ 0 & -1 & 0 & | & -2 \\ 0 & -5 & 4 & | & 2 \end{bmatrix} \xrightarrow{-1R_2 \to R_2} \begin{bmatrix} 1 & 1 & -1 & | & 0 \\ 0 & 1 & 0 & | & 2 \\ 0 & -5 & 4 & | & 2 \end{bmatrix}$$

To get 0 in row 3 below the 1 in row 2, perform the operation: $5R_2 + R_3 \to R_3$

$$\begin{bmatrix} 1 & 1 & -1 & | & 0 \\ 0 & 1 & 0 & | & 2 \\ 0 & -5 & 4 & | & 2 \end{bmatrix} \xrightarrow{5R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 1 & -1 & | & 0 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 4 & | & 12 \end{bmatrix}$$

Step 5: Perform row operations to get a 1 in row 3, column 3.

$$\begin{bmatrix} 1 & 1 & -1 & | & 0 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 4 & | & 12 \end{bmatrix} \xrightarrow{\frac{1}{4}R_3 \to R_3} \begin{bmatrix} 1 & 1 & -1 & | & 0 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

Step 6: The obtained matrix is the <u>row echelon</u> form of the augmented matrix. Solve the system corresponding to it using back-substitution.

$$\begin{cases} x + y - z = 0 \\ \quad y \quad = 2 \\ \qquad z = 3 \end{cases}$$

Replace y and z in the first equation with their values:

$x + 2 - 3 = 0 \qquad x = 1$ **Answer:** $\{(1,2,3)\}$

**17.** Solve the system $\begin{cases} 3x - 2y + z = 2 \\ x + y - z \quad = 0 \\ 2x + y - 2z = -2 \end{cases}$ to the <u>reduced row-echelon form</u> (Gauss-Jordan elimination)

**Answer:** $\begin{bmatrix} 1 & 0 & 0|1 \\ 0 & 1 & 0|2 \\ 0 & 0 & 1|3 \end{bmatrix}$, $\{(1,2,3)\}$

**18.** Determine whether the system is consistent. If the system is consistent, solve it.

$$\begin{cases} 3x - 2y + z = 2 \\ x + y - z = 0 \\ 4x - y = 2 \end{cases}$$

**Solution**

Reduce the augmented matrix to row-echelon form or to reduced row-echelon form.

$$\begin{bmatrix} 3 & -2 & 1|2 \\ 1 & 1 & -1|0 \\ 4 & -1 & 0|2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 1 & -1|0 \\ 3 & -2 & 1|\ 2 \\ 4 & -1 & 0|\ 2 \end{bmatrix} \xrightarrow{R_2 = -3r_1 + r_2} \begin{bmatrix} 1 & 1 & -1|0 \\ 0 & -5 & 4|\ 2 \\ 4 & -1 & 0|\ 2 \end{bmatrix}$$

$$\xrightarrow{R_3 = -4r_1 + r_3} \begin{bmatrix} 1 & 1 & -1|0 \\ 0 & -5 & 4|\ 2 \\ 0 & -5 & 4|\ 2 \end{bmatrix} \xrightarrow{R_3 = -r_2 + r_3} \begin{bmatrix} 1 & 1 & -1|0 \\ 0 & -5 & 4|\ 2 \\ 0 & 0 & 0|\ 0 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 1 & -1 \vdots 0 \\ 0 & -5 & 4 \vdots 2 \end{bmatrix} \xrightarrow{R_2 = -1/5\, r_2} \begin{bmatrix} 1 & 1 & -1 & \vdots & 0 \\ 0 & 1 & -4/5 & \vdots & -2/5 \end{bmatrix}$$

The system is consistent. The last augmented matrix corresponds to the system:

$$\begin{cases} x + y - z = 0 \\ y - \frac{4}{5}z = -\frac{2}{5} \end{cases}$$

Solve the last equation for $y$:

$$y = \frac{4}{5}z - \frac{2}{5}$$

Substitute the expression for $y$ in the first equation:

$$x + (\frac{4}{5}z - \frac{2}{5}) - z = 0$$

Combine like terms and solve the equation for $x$:

$$x = \frac{1}{5}z + \frac{2}{5}$$

**Answer:** consistent, infinitely many solutions: $\{(x, y, z)|(x = \frac{1}{5}z + \frac{2}{5}, y = \frac{4}{5}z - \frac{2}{5},\ z\epsilon R\}$, dependent equations

**19.** Solve the inequality: $x^3 - x^2 - x + 1 > 0$.
**Answer:** $(-1,1) \cup (1, \infty)$

**20.** Solve the inequality: $x - \frac{2}{x-1} \leq 0$.
**Answer:** $(-\infty, -1] \cup (-1,2]$