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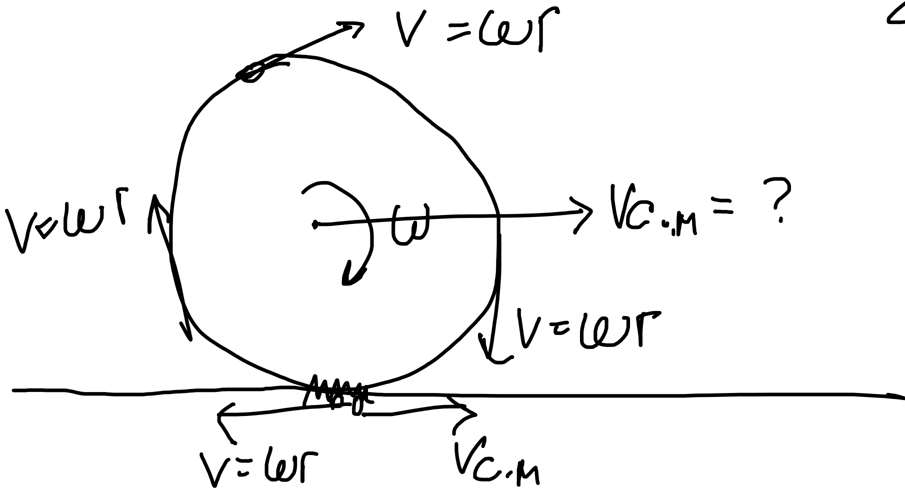
Rolling w/o slipping

$V_{CM} = V_{RIM} = \omega r$



19. A uniform meter stick is held vertically with one end on the floor and is then allowed to fall. Find the speed of the other end just before it hits the floor, assuming that the end on the floor does not slip.

Ans. 5.42 m/s

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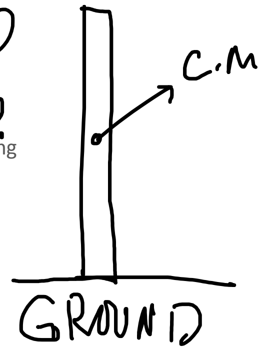
$K = 0$

$U = ?$

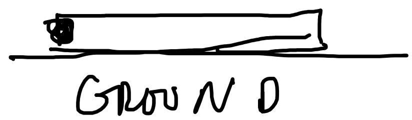
$mg\left(\frac{L}{2}\right)$



$I = \frac{1}{3} mL^2$

$K = \frac{1}{2} I\omega^2$

$U = 0$



$$0 + mg\frac{L}{2} = \frac{1}{2}\underbrace{I}_{\frac{1}{3}mL^2}\omega^2 + 0$$

$$\cancel{m}g\frac{\cancel{L}}{\cancel{2}} = \frac{1}{\cancel{2}}\left(\frac{1}{3}\cancel{m}L^{\cancel{2}}\right)\omega^2$$

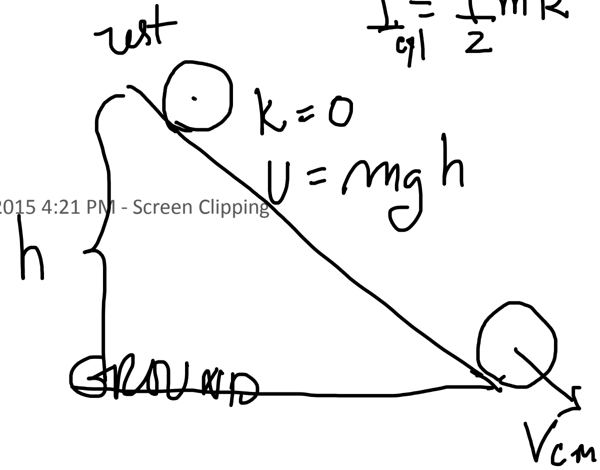
$$g = \frac{1}{3}L\omega^2 \qquad \omega = \sqrt{\frac{3g}{L}} \quad g \to 9.8,\ L \to 1$$

$$v = \omega \cdot L = 5.42 \text{ m/s} \qquad \omega =$$

22. A uniform cylinder rolls down a plane inclined at an angle θ with the horizontal. Show that if the cylinder rolls without slipping, the acceleration is $a = \frac{2}{3} g \sin(\theta)$.

a) $V = ?$ at the bottom

$I_{cyl} = \frac{1}{2} m R^2$



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$K = \frac{1}{2} m V_{cm}^2 + \frac{1}{2} I \omega^2$

$U = 0$

$$0 + mgh = \frac{1}{2} m \overset{?}{V}_{cm}^2 + \frac{1}{2}\left[\frac{1}{2} m R^2\right] \overset{?}{\omega}^2$$

w/o slipping

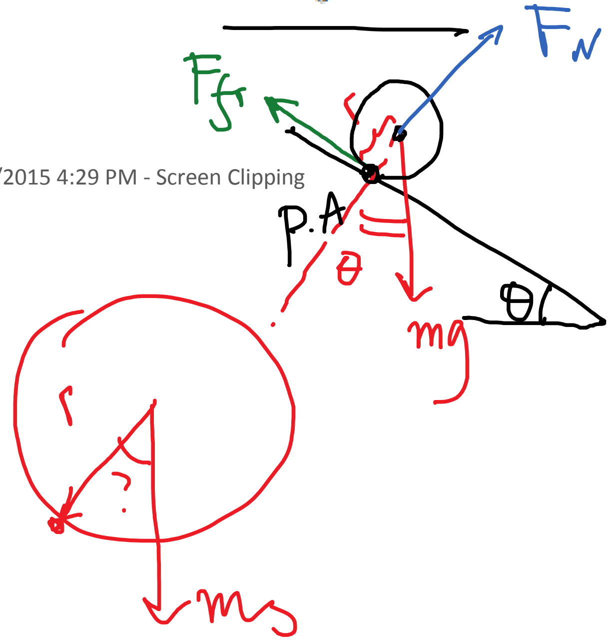
$V_{cm} = \omega R$

$$\cancel{m}gh = \frac{1}{2} \cancel{m} V_{c.m}^2 + \frac{1}{2}\left[\frac{1}{2} \cancel{m} \cancel{R^2}\right] \frac{V_{cm}^2}{\cancel{R^2}}$$

$$gh = \frac{1}{2} V_{c.m}^2 + \frac{1}{4} V_{cm}^2 = \frac{3}{4} V_{cm}^2$$

$$V_{c.m} = \sqrt{\frac{4gh}{3}}$$

22. A uniform cylinder rolls down a plane inclined at an angle θ with the horizontal. Show that if the cylinder rolls without slipping, the acceleration is $a = \frac{2}{3} g \sin(\theta)$.



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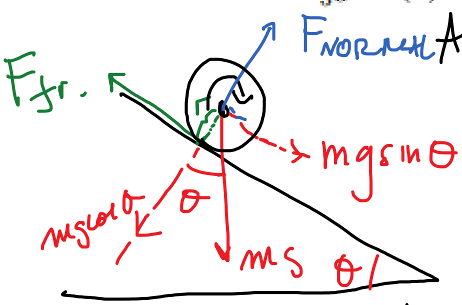
FIND $a = ?$

TORQUE

$$\tau_{mg} = (mg)(r)\sin\theta$$

$$\tau_{NET} = I\alpha$$

22. A uniform cylinder rolls down a plane inclined at an angle θ with the horizontal. Show that if the cylinder rolls without slipping, the acceleration is $a = \frac{2}{3} g \sin(\theta)$.



About C-M

$\tau_{F_N} \equiv 0$

$\tau_{mg} \equiv 0$

$\tau_{F_{fr}} \equiv (F_{fr})(r)\sin 90$

$F_{NET} = ma$

$$\boxed{mg\sin\theta - F_{fr} = ma}$$

w/o slipping

$$\|\tau_{NET}\| = \|r F_{fr}\| = \|I \alpha\|$$

$I = \frac{1}{2}mr^2$, $\alpha = \frac{a}{r}$

$$\cancel{r} F_{fr} = \frac{1}{2} m \cancel{r^2} \frac{a}{\cancel{r}}$$

$$\boxed{F_{fr} = \frac{1}{2} ma}$$

F: $mg\sin\theta - F_{fr} = ma$

τ: $F_{fr} = \frac{1}{2} ma$

$$\cancel{m}g\sin\theta - \frac{1}{2}\cancel{m}a = \cancel{m}a$$

$$g\sin\theta = \frac{3}{2}a$$

$$a = \frac{2g\sin\theta}{3}$$

EXAMPLE

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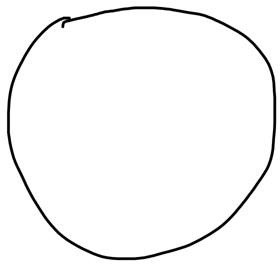
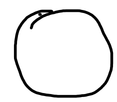
COLLAPSING STAR





$$I_1\omega_1 = I_2\omega_2$$

$M_2 = 2\times M_{SUN}$

$M_1 = 2\times M_{SUN}$

$R_2 = 10$ km

$R_1 = 7\times 10^5$ km

$\omega_1 = 1.0$ revolution every 10days

$\omega_2 = ?$

$$\left(\frac{2}{\cancel{5}}\cancel{M}R_1^2\right)\omega_1 = \left(\frac{2}{\cancel{5}}\cancel{M}R_2^2\right)(\omega_2)$$

$$\frac{1\,\text{rev}}{10\,\text{day}} = \frac{1\,\text{rev}}{10(24\,\text{h})(60\,\text{min})} = 6.94\times10^{-4}\,\text{rpm}$$

$$(7\times10^5\,\cancel{\text{km}})^2(6.94\times10^{-4}\,\text{rpm}) = (10\,\cancel{\text{km}})^2\omega_2$$

$$\omega_2 = \left(\frac{7\times10^5}{10}\right)^2 6.94\times10^{-4}\,\text{rpm}$$

$$\omega_2 = 3.4\times10^6\,\frac{\text{rev}}{\text{min}} = 5.6\times10^4\,\frac{\text{rev}}{\text{s}}$$