3. The angular position of a point on a rotating wheel is given by the function $\theta = 2.0 + 4.0t^2 + 2.0t^3$, where θ is in radians and t in seconds. At $t = 0$, what are

   (a) the point's angular position and $\Rightarrow$ $t=0 \quad \theta = 2.0 \frac{rad}{s}$

   (b) its angular velocity? $\Rightarrow$ $\omega = \frac{d\theta}{dt} = 8t + 6t^2 \xrightarrow[t \to 0]{} 0 \frac{rad}{s}$

   (c) What is its angular velocity at $t = 4.0$ s? $\rightarrow$ $t = 4s \quad \omega = 8(4) + 6(4)^2 = 128 \frac{rad}{s}$

   (d) Calculate it angular acceleration at $t = 2.0$ s. $\rightarrow$ $\alpha = \frac{d\omega}{dt} = 8 + 12t \xrightarrow[t=2.0s]{} 32 \frac{rad}{s^2}$

   (e) Is its angular acceleration constant?

Ans. (a) 2.0 rad (b) $0.0\frac{\text{rad}}{\text{s}}$ (c) $128\frac{\text{rad}}{\text{s}}$ (d) $32\frac{\text{rad}}{\text{s}^2}$

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5. The angular acceleration of a wheel, as a function of time, is $\alpha = 5.0t^2 - 3.5t$, where α is in rad/s^2 and t in seconds. If the wheel starts from rest ($\theta = 0$ and $\omega = 0$ at $t = 0$), determine

   (a) the angular velocity ω, $\Rightarrow$ $\omega = \int \alpha\, dt = \int (5t^2 - 3.5t)\, dt = \frac{5t^3}{3} - \frac{3.5t^2}{2} + \text{const}$

   (b) and the angular position θ as a function of time.

   (c) Evaluate ω and θ at $t = 2.0$ s.

$t = 0 \quad \omega = 0 \Rightarrow \text{const} \equiv 0$

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$\theta = \int \omega\, dt = \int \left(\frac{5t^3}{3} - \frac{3.5t^2}{2}\right) dt$

$$\boxed{\omega = \frac{5t^3}{3} - \frac{3.5t^2}{2}}$$

6. Starting from rest, a disk rotates about its central axis with constant angular acceleration. In 5.0 s, it rotates 25 rad. During that time, what are the magnitudes of

   (a) the angular acceleration and the average velocity?

   (b) What is the instantaneous angular velocity of the disk at the end of the 5.0 s?

   (c) With the angular acceleration unchanged, through what additional angle will the disk turn during the next 5.0 s?

Ans. (a) $2.0\frac{\text{rad}}{\text{s}^2}, 5.0\frac{\text{rad}}{\text{s}}$; (b) $10\frac{\text{rad}}{\text{s}}$; (c) 75 rad

Given

$\omega_0 = 0$

$\alpha = \text{const}$

$\Delta t = 5s$

$\Delta\theta = 25\,\text{rad}$

a) $\alpha = ?$ $\omega_{av} = ?$

$$\theta = \theta_0 + \omega_0 \Delta t + \frac{\alpha \Delta t^2}{2}$$

$$\underbrace{\Delta\theta}_{25\,\text{rad}} = (0)(5s) + \alpha\frac{(5s)^2}{2} \Rightarrow \alpha = 2.0\frac{\text{rad}}{s^2}$$

$$\omega_{av.} = \frac{\Delta\theta}{\Delta t} = \frac{25\,\text{rad}}{5s} = 5\frac{\text{rad}}{s}$$

b) $\omega = ?$

$$\omega = \omega_0 + \alpha\Delta t = 0 + \left(2.0\frac{\text{rad}}{s^2}\right)(5s) = 10\frac{\text{rad}}{s}$$

c) $\Delta\theta = ?$ $\Delta t = 10s$

$$\Delta\theta = \cancel{\omega_0 \Delta t}^{\,0} + \frac{\alpha\Delta t^2}{2} = \frac{\left(2.0\frac{\text{rad}}{s^2}\right)(10s)^2}{2}$$

$$\Delta\theta = 100\frac{\text{rad}}{s} \Rightarrow \text{additional } 75\frac{\text{rad}}{s}$$

10. A 0.84-m diameter solid sphere can be rotated about an axis through its center by a torque of 10.8 m.N which accelerates it uniformly from rest through a total of 180 revolutions in 15.0 s. What is the mass of the sphere?

Ans. 15 kg

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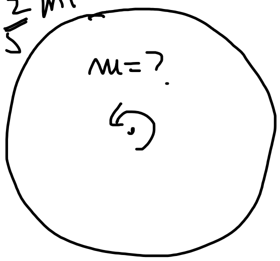
$I = \frac{2}{5} mR^2$



Solid sphere

$R = 0.42\ m$

$\tau = 10.8\ m.N$

$\omega_0 = 0$

$\Delta\theta = 180\ rev.$

$\Delta t = 15.0 s$

$\tau = I \alpha$

$\tau \to 10.8$, $I \to ? \to m$

$\cancel{\theta = \theta_0} + \cancel{\omega_0 \Delta t}^{\,0} + \frac{\alpha \Delta t^2}{2}$

$(180)(2\pi) = \alpha \frac{(15)^2}{2}$

$\alpha = 10.048\ \frac{rad}{s^2}$

$I = \frac{\tau}{\alpha} = \frac{10.8}{10.048}$

$\frac{2}{5} mR^2 = I$

$m =$

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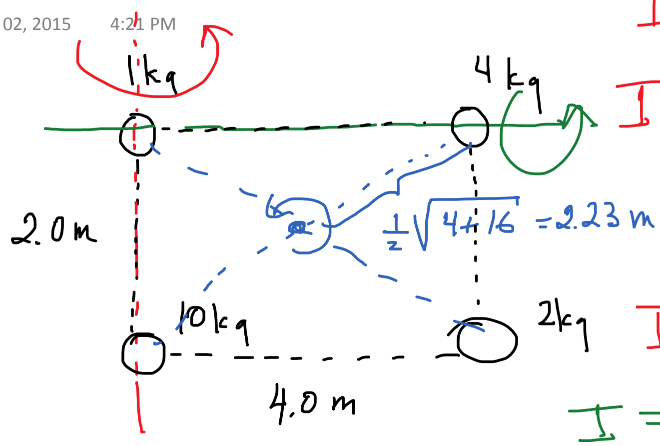

$I = ?$

$I = (1kg)(\emptyset)^2 + (4kg)(4m)^2 + (2\,kg)(4.0m)^2 + (10kg)\,0^2$

$I = 96\ kg \cdot m^2$

$I = (1kg)\,\emptyset^2 + (4kg)\,0^2 + (10\,kg)(2)^2 + (2\,kg)(2.0)^2$

$I = 48\ kg \cdot m^2$

$I = (1kg)\,(2.23m)^2 + (4kg)\,(2.23\,m)^2 + (2\,kg)(2.23\,m)^2 + (10kg)\,(2.23)^2$

Parallel Axis Theorem

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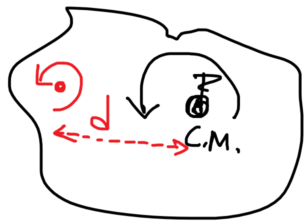
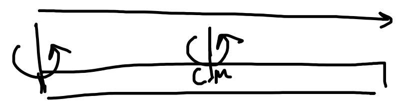


$I_{C.M}$

1) Parallel

$$I = I_{C.M} + md^2$$

Example



$$I_{C.M} = \frac{1}{12} mL^2$$

$$I = \frac{1}{12} mL^2 + m\left(\frac{1}{2} L\right)^2$$

$$\left(\frac{1}{12} + \frac{1}{4}\right) mL^2 = \frac{1}{3} mL^2$$

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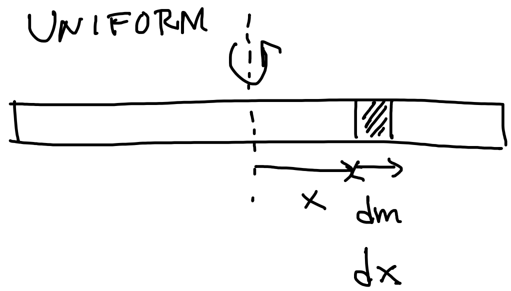

m, L

$$dI = dm \cdot x^2$$

$$\frac{dm}{M} = \frac{dx}{L} \qquad dm = \frac{M}{L}dx$$

$$I = \int dI = \int_{x=-\frac{L}{2}}^{+\frac{L}{2}} \frac{M}{L}dx \cdot x^2 = \frac{M}{L}\int_{-\frac{L}{2}}^{L/2} x^2 dx = \frac{M}{L}\frac{x^3}{3}\Bigg|_{-\frac{L}{2}}^{+\frac{L}{2}}$$

$$I = \frac{1}{3}\frac{M}{L}\left(\frac{L^3}{8} - \left(\frac{-L^3}{8}\right)\right)$$

$$\frac{1}{3}\frac{M}{\cancel{L}}\frac{\cancel{2}L^{\cancel{3}}}{\cancel{8}\,4} = \frac{1}{12}ML^2$$

Rotational Inertia

RING

DISK

https://www.youtube.com/watch?v=_4Fg7yeuAGc https://www.youtube.com/watch?v=EvxIhrToJnA

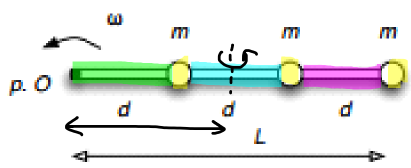

Figure 10.1: Rotating Particles (Problem 11)

Balls $I = md^2 + m(2d)^2 + m(3d)^2$

$I = \underline{\underline{14md^2}}$

11. Figure 10.1 shows three 0.010 kg particles that have been glued to a rod of length $L = 6.0$ cm and negligible mass. The assembly can rotate around a perpendicular axis through point O at the left end. Determine the moment of inertia of the assembly around the rotation axis.

Ans. $14md^2$

FOLLOW UP THE MASS OF RODS IS $M = 0.2\,kg$

GREEN: $I = \frac{1}{3}Md^2$

BLUE: $I = \left(\frac{1}{12}Md^2\right) + M\left(\frac{3}{2}d\right)^2$

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