Northern Virginia Community College
Fifth Annual NOVA Mathematics Competition
April 7, 2017
Competition Questions
Round 1
Question 1 to 10
Question 1

Given a natural number $n$, find all possible real values $r$ so that multiplying $r^n$ by $r$ gives the same result as dividing $r^n$ by $r$.

The solution =

Official Use Only: Is answer is correct? Yes  No
Question 2

Team Number: __________

Convert a number $B8DA$ (base 16) into (base 6).

The solution =

Official Use Only: Is answer is correct? Yes No
Question 3

Find the exact value of \( \sum_{n=2}^{100} \left( \frac{1}{n} - \frac{1}{n+2} \right) \)

(Answer in the form of a simplified fraction)

The solution =

Official Use Only: Is answer correct? Yes No
The number $K$ is chosen at random from the set of all of the 5-digit numbers that have the sum of their digits equal to 41. What is the probability that $K$ is divisible by 11?

The solution =

Official Use Only: Is answer correct? Yes No
Two types of vehicle may be used to get to the airport: cars that each hold up to 5 passengers and charge for $29 a trip to the airport, and vans that each hold up to 7 passengers and charge $41. Determine the minimum cost to transport 49 people to the airport.

The solution =

Official Use Only: Is answer correct?  Yes  No
Let \( f(x) = x^2 + bx + c \). If \( f(4) = f(2) + 11 \), find \( f(4) - f(0) \).
Question 7

How many positive integers less than 1000 are divisible by 7 or 11, but not both?

The solution =

Official Use Only : Is answer is correct? Yes No
Question 8

How many natural numbers are there in the set of common factors of 440 and 680?

The solution =

Official Use Only: Is answer correct? Yes No
Question 9

Team Number: ____________

If the number \( D \) has (base 15) representation of \( abc \) and (base 6) representation of \( 1abc \), find \( a + b \).

The solution =

Official Use Only : Is answer is correct? Yes No
Question 10

How many ordered pairs \((k, n)\) of integers are solutions to \(k^2 - 2016 = 3^n\)?

The solution =

Official Use Only: Is answer is correct? Yes No
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Competition Questions
Round 2
Question 11 to 20
Question 11

If a circle is inscribed in an equilateral triangle of side $S$. What is the ratio (expressed as a decimal rounded to the nearest hundredth) of the area of a circle to the area of the triangle?

The solution =

Official Use Only : Is answer is correct? Yes No
Question 12

Alice takes her favorite number, adds 5 to it, multiplies the result by 10, subtracts 20 from the result, and then drops the final digit—which happens to be zero. If Alice’s (correct) answer is 9, what is her favorite number?

The solution =

Official Use Only: Is answer is correct? Yes No
In my town, it’s rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability $\frac{1}{2}$, and given that it is not rainy, there will be heavy traffic with probability $\frac{1}{4}$. If it’s rainy and there is heavy traffic, I arrive late for work with probability $\frac{1}{2}$. On the other hand, the probability of being late is reduced to $\frac{1}{8}$ if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is $\frac{1}{4}$. You pick a random day. What is the probability that it’s not raining and there is heavy traffic and I am not late?

The solution =
Question 14

Andre and Bill stand back to back: at a distant 0 from each other. At exactly 2:00:17 PM on the clock, they begin to walk away from one another. Andre walks North at the rate of 4 ft/sec and Bill walks East at the rate of 6 ft/sec. What time is it on the clock when the two men are 114 ft away from one another?

The solution =

Official Use Only : Is answer is correct? Yes No
Solve the following (each letter is a distinct digit); evaluate DJT.

DJT = (D + J + T) \times D \times J \times T

The solution =

Official Use Only : Is answer is correct?   Yes   No
Question 16

Suppose that the two 3-digit numbers $abc$ and $cba$ differ by 792 \((c, a \neq 0)\). If the middle digit $b = 6$, find the sum of the two numbers.

The solution =

| Official Use Only : Is answer is correct? | Yes | No |
Question 17

A rectangle is 4 times as long as it is wide. If the length is increased by 4 inches and the width is decreased by 1 inch, the area will be 60 square inches. What were the dimensions of the original rectangle?

The solution =

Official Use Only: Is answer is correct?  Yes  No
Question 18

Find all possible natural numbers $n$ for which $n^4 + 4^n$ is a prime.

The solution =

Official Use Only: Is answer correct? Yes No
Question 19

How many prime numbers less than 100 are there?

The solution =

Official Use Only: Is answer correct? Yes No
Question 20

Find all possible exact value(s) of $A$ such that

$$\csc(6A + \frac{\pi}{8}) = \sec(2A + \frac{\pi}{8}).$$

Where $0 < A < \frac{\pi}{4}$

The solution =

Official Use Only: Is answer correct?  Yes  No