Integration Formula Sheet for Calculus 2

Created by MSW
http://www.nvcc.edu/home/mwesterhoff

Basic Integration Formulas

$$\int 0\,du = 0$$

$$\int du = u + C$$

$$\int k\,du = ku + C$$

$$\int u^n\,du = \frac{u^{n+1}}{n+1} \qquad (\text{for } n \neq -1)$$

$$\int \cos u\,du = \sin u + C$$

$$\int \sin u\,du = -\cos u + C$$

$$\int \sec^2 u\,du = \tan u + C$$

$$\int \sec u \tan u\;du = \sec u + C$$

$$\int \csc^2 u\,du = -\cot u + C$$

$$\int \csc u \cot u\;du = -\csc u + C$$

$$\int e^u\,du = e^u + C$$

$$\int a^u\,du = \left(\frac{1}{\ln a}\right) a^u + C, a > 0$$

$$\int \frac{1}{u}\,du = \ln|u| + C$$

$$\int \tan u\,du = -\ln|\cos x| + C = \ln|\sec u| + C$$

$$\int \cot u\,du = \ln|\sin u| + C = -\ln|\csc u| + C$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C = -\ln|\sec u - \tan u| + C$$

$$\int \csc u\,du = -\ln|\csc u + \cot u| + C = \ln|\csc u - \cot u| + C$$

Integration Formulas

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\text{arcsec}\frac{|u|}{a} + C$$

Summation Formulas

$$\sum_{i=1}^{n} = a_1 + a_2 + \ldots + a_n$$

$$\sum_{i=1}^{n} c = cn$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=1}^{n} i^4 = \frac{n(2n+1)(n+1)(3n^2+3n-1)}{30}$$

Upper and Lower Sums

I.R. $= \lim_{n\to\infty}\sum_{i=1}^{n} f(m_i)\Delta x$

$f(m_i)$ is the min. value of f on the subinterval.

C.R. $= \lim_{n\to\infty}\sum_{i=1}^{n} f(M_i)\Delta x$

$f(M_i)$ is the max. value of f on the subinterval.

Left Endpoints: $a + (i-1)\Delta x$, for $i = 1, \ldots, n$

Right Endpoints: $a + i\Delta x$, for $i = 1, \ldots, n$
$\Delta x = (b-a)/n$

Definite (Riemann) Integral

$$\lim_{||\Delta\to 0||}\sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)\,dx$$

Partitions of equal width: $||\Delta|| = \Delta x$

Partitions of unequal width: $||\Delta|| = \max(\Delta x_i)$

Continuity $\Rightarrow$ Integrability
The Converse is NOT True (see example below)

Example: $\int_0^2 \lfloor x \rfloor\,dx = 1$

Area of a Region in a Plane

Area of the region bounded by the graph of f, x-axis, and $x = a$ and $x = b$

Area $= \lim_{n\to\infty}\sum_{i=1}^{n} f(r_i)\Delta x = \int_a^b f(x)\,dx$
where $r_i \in [x_{i-1}, x_i]$

Properties of Integration

1. $\int_a^a f(x)\,dx = 0$
2. $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
3. $\int_a^b f(x)\,dx = \int_a^c f(x) + \int_c^b f(x)\,dx$
4. $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$
5. $\int [f(x)dx \pm g(x) \pm \cdots \pm k(x)]\;dx = \int_a^b f(x) \pm \int_a^b g(x)\,dx \pm \cdots \pm \int_a^b k(x)\,dx$
6. $0 \leq \int_a^b f(x)\,dx$ for f non-negative
7. If $f(x) \leq g(x) \Rightarrow \int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$
8. If f is an even function, then $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$
9. If f is an odd function, then $\int_{-a}^a f(x)\,dx = 0$

Fundamental Theorem of Calculus I (FTC I)

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Fundamental Theorem of Calculus II (FTC II)

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

$$\frac{d}{dx}\left[\int_a^u f(t)\,dt\right] = f(u)u' \text{ (with chain rule)}$$

Mean Value Theorem for Integration

$\int_a^b f(x)\,dx = f(c)(b-a)$, where $c \in [a,b]$

Average Value of a Function on $[a,b]$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Substitution Method for Integration

Let $u = g(x)$, then $\frac{du}{dx} = g'(x) \Rightarrow du = g'(x)dx$,

$$\int_a^b f(g(x))g'(x)\,dx \Rightarrow \int_a^b f(u)\,du \Rightarrow \int_{g(a)}^{g(b)} f(u)\,du$$

Numerical Integration

Midpoint Rule: $f(x)\,dx \approx \sum_{i=1}^{n} f\left(\frac{x_i + x_{i-1}}{2}\right)\Delta x$

Trapezoidal Rule: $\int_a^b f(x)\,dx \approx \sum_{i=1}^{n}\left(\frac{f(x_i)+f(x_{i-1})}{2}\right)\Delta x$

Simpson's Rule: $\int_a^b f(x)\,dx \approx \frac{b-a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)]$