

Total points: 40. Total time: 1.5 hr. Show all your work.

You must write down your answers with units in the Ans. boxes

1. (6 pt) Determine the conservative force if its potential energy is given by:

$$U(x) = \cos(ax) + e^{-bx} ,$$

where a and b are constants.

Def. $\Delta U = -W = - \int F dx \Rightarrow F = -\frac{dU}{dx}$

$$F = -\frac{d}{dx} [\cos ax + e^{-bx}] = -(-a \sin ax - be^{-bx})$$

Ans. $a \sin ax + be^{-bx}$

2. (4pt) As a 2.0-kg object moves from $\vec{r}_1 = (-7\hat{i} + 2\hat{j})$ m to $\vec{r}_2 = (8\hat{i} + 4\hat{j})$ m, the constant resultant force acting on it is equal to $\vec{F} = (2\hat{i} - 3\hat{j})$ N. If the speed of the the object at the initial position is 4.0 m/s, what is its kinetic energy at its final position?

$$\vec{d} = \vec{r}_2 - \vec{r}_1 = [8 - (-7)]\hat{i} + (4 - 2)\hat{j} = 15\hat{i} + 2\hat{j}$$

$$W = \vec{F} \cdot \vec{d} = [2\hat{i} - 3\hat{j}] \cdot [15\hat{i} + 2\hat{j}] = 2(15) + (-3)(2) = 24$$

$$\Delta K = W \Rightarrow K_f = K_0 + W = 16 + 24 = 40$$

Ans. 40 J

3. (4 pt) In a given displacement of a particle, its kinetic energy increases by 30 J while its potential energy decreases by 10 J. Determine the work of the nonconservative forces acting on the particle during this displacement.

$$\Delta E = W_{\text{non conservative}}$$

$$\Delta E = \Delta K + \Delta U = +30 \text{ J} + (-10 \text{ J}) = 20 \text{ J}$$

Ans. 20 J

4. (6 pt) A 12-kg projectile is launched directly upward with an initial vertical speed of 20 m/s. It rises to a maximum height of 18 m above the launch point. How much work is done by the dissipative resistive force (air resistance) on the projectile during this ascent?

$$\vec{E}_{\text{TOP}} = K + U = 0 + 12(9.8)(18) = 2116.8 \text{ J}$$

Ans.

$$W = \Delta E = 2116.8 - 2400 = -283.2 \text{ J}$$

$$\vec{E}_{\text{Bot}} = K + U = \frac{1}{2}(12)(20)^2 + 0 = 2400 \text{ J}$$

TOT. MASS
1.3

5. (6 pt) Three odd-shaped blocks of chocolate have the following masses and center-of-mass coordinates: (a) 0.300 kg; (0.200 m, 0.300 m) (b) 0.700 kg; (0.400 m, -0.100 m) (c) 0.300 kg; (-0.300 m, 0.600 m) Find the y-coordinate of the center of mass of the system of three chocolate blocks.

$$\frac{0.3}{1.3}(0.3) + \frac{0.7}{1.3}(-0.1) + \frac{0.3}{1.3}(0.6) =$$

Ans. 0.154 m

6. (4 pt) You are standing on a large sheet of frictionless ice and are holding a large rock. In order to get off the ice, you throw the rock directly horizontally so it has velocity relative to the earth of 12.0 m/s. If your mass is 70.0 kg and the rock's mass is 15.0 kg, what is your speed after you throw the rock?

Before: 0

$$v = \frac{-(15)(12)}{70} =$$

After: $(15 \text{ kg})(12.0 \frac{\text{m}}{\text{s}}) + (70 \text{ kg})(v)$

Ans. 2.57 $\frac{\text{m}}{\text{s}}$ in opposite direction

7. (6 pt) A 4.0-kg object moving with a velocity of 7.0 m/s in the positive x direction strikes and sticks to a 3.0-kg object moving initially with a speed of 2.0 m/s in the opposite direction. How much kinetic energy is lost in this collision?

$$(4.0 \text{ kg})(7.0 \frac{\text{m}}{\text{s}}) + (3.0 \text{ kg})(-2.0 \frac{\text{m}}{\text{s}}) = (7.0 \text{ kg})v \Rightarrow v = 3.14 \frac{\text{m}}{\text{s}}$$

$$\Delta K = \frac{1}{2}(7.0)(3.14)^2 - \left[\frac{1}{2}4(7)^2 + \frac{1}{2}3(2)^2 \right] = -27.5 \text{ J}$$

Ans.

8. (4 pt) A 2.0-kg block sliding on a rough horizontal surface is attached to one end of a horizontal spring ($k = 250 \text{ N/m}$) which has its other end fixed. The block passes through the equilibrium position with a speed of 2.6 m/s and first comes to rest at a displacement of 0.20 m from equilibrium. What is the coefficient of kinetic friction between the block and the horizontal surface?

$x=0 \quad v=0 \quad K = \frac{1}{2}(2)(2.6)^2 = 6.76 \text{ J}$

Ans. $\mu F_N = \mu mg$

$\Delta E = 5 - 6.76 = -1.76 \text{ J} = W_{fr} = F_{fr} \cdot d \cos(180)$

$x=0.2 \quad U = \frac{1}{2}(250)(0.2)^2 = 5.0 \text{ J}$

$K=0$

$-1.76 = \mu(2.0)(9.8)(0.2) \Rightarrow \mu = 0.45$

The diagram shows two scenarios. The top part shows a block attached to a spring on a smooth surface, with a coordinate system where x=0 is the equilibrium position. The bottom part shows a block on a rough horizontal surface, with a coordinate system where x=0.2 is the point where the block comes to rest.