Newton's Second Law

Lab Report
September 17, 2012

1 Goal

The goal of the experiment is to determine the acceleration along a frictionless horizontal ramp of a glider that has attached to it a rope from which a hanging mass is suspended.

2 Theory

The main theory behind this experiment is Newton's Second Law, which defines the relationship between force and the acceleration a force produces. Newton's second law states that the force on an object is directly proportional to its acceleration when the mass is constant. It also states that when force is constant, acceleration is inversely proportional to the mass of the object.

These properties are defined by the following mathematical equation:

$$\vec{\boldsymbol{F}} = \boldsymbol{m}\vec{\boldsymbol{a}} \tag{1}$$

where $\vec{\boldsymbol{F}}$ is the net force acting on the object and $\boldsymbol{m}$ is to total mass of the object being accelerated.

In order to test Newton's second law, we placed a glider (m_1) upon a frictionless air track from which we suspended a mass (m_2) on a string through a pulley. Attached to the glider was a picket fence designed to go through a photogate which was connected to a computer.

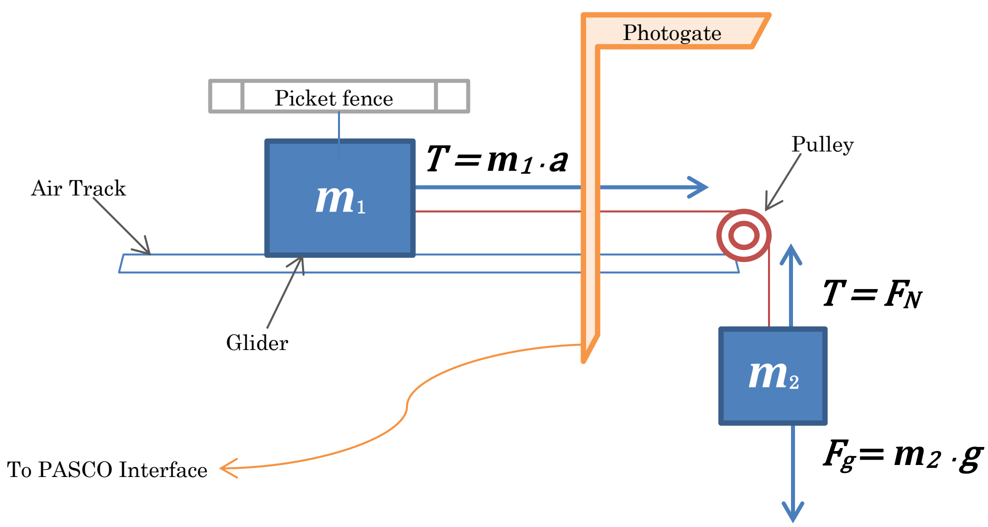

Figure 1: Air Track Setup

The total force of our system is the addition of the horizontal force (F_x) acting on the glider and the net vertical force (F_y) acting on the suspended mass.

The only horizontal force is the tension (T) in the rope. Applying Newton's second law, the tension is the mass of the glider times the acceleration, which can be represented as

$$\vec{F_x} = T = m_1\vec{a} \tag{2}$$

The net vertical force is the gravitational force (Fg = suspended mass · gravitational force, $\vec{\boldsymbol{g}}$) minus the normal force (F_N), which is the same as tension (T). Using Newton's second law, this net vertical force can be represented as:

$$\vec{F_y} = F_g - F_N = m_2 \cdot \vec{a}$$

$$\vec{F_y} = m_2 \cdot g - T = m_2 \cdot \vec{a} \tag{3}$$

Adding equations (2) and (3), acceleration can then be calculated as:

$$a = \frac{m_2 \cdot g}{(m_1 + m_2)} \quad , \quad g = 9.81 \pm .01 \left[\frac{\text{m}}{\text{s}^2}\right] \tag{4}$$

3 Experimental Settings

The setup and forces in our experiment are shown on the diagram in Figure 1. The procedure for our experiment was as follows:

1. We leveled the track first, by adjusting its supports and adding additional sheets of paper to level the track.

2. We adjusted the photogate's height to ensure that it will "blink" every time a marked section of the picket fence passed through it.

3. The photogate was connected to a computer through PASCO interface. We used the Data Studio program to analyze the signals from the photogate.

4. We weighed the glider on a scale. We also weighed three different sets of weights to approximate 4g, 6g and 8g of mass which we then attached to the end of our string for each of our three runs.

5. In the Data Studio, we plotted the graph "velocity vs. time" after each run and recorded the slope of that graph. That was our "measured" acceleration.

4 Data and Results

m_1 [g]	m_2 [g]	$a^{theoretical}\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$	$a^{measured}\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$	$\frac{\|a^{meas}-a^{th}\|}{a^{th}}$
316.50 ± 0.01	3.88 ± 0.01	0.119 ± <0.001	0.120 ± 0.0018	0.84 %
316.50 ± 0.01	5.79 ± 0.01	0.176 ± <0.001	0.172 ± 0.0024	2.30 %
316.50 ± 0.01	7.75 ± 0.01	0.234 ± <0.001	0.234 ± 0.0033	0.21 %

Table 1: Data and Results

Our data and results are given in the table above. The last column gives the relative (percent) difference between the two values for acceleration, a^{th} and a^{meas}. The uncertainty of the measurements, m_1 and m_2, were determined by the scale used. The uncertainty of the measured acceleration was taken equal to the uncertainty in the slope of the best linear fit of the velocity vs. time graph. The uncertainty of the theoretical value for the acceleration was calculated based on the formula for acceleration:

$$a^{\mathrm{th}} = \frac{m_2 \cdot g}{(m_1 + m_2)}\ .$$

The relative uncertainty of the theoretical acceleration can then be calculated as:

$$\frac{\Delta a^{\mathrm{th}}}{a^{\mathrm{th}}} = \frac{\Delta m_2}{m_2} + \frac{\Delta(m_2 + m_1)}{(m_2 + m_1)} + \frac{\Delta g}{g}\ ,$$

where we have used 9.81 ± 0.01 $\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$ for the value of the gravitational acceleration.

The relative uncertainty of the measured acceleration is :

$$\frac{\Delta a^{\mathrm{meas}}}{a^{\mathrm{meas}}}\ .$$

The relative uncertainty for theoretical and measured acceleration for each run is shown in Table 2 below.

m_2 [g]	$a^{theoretical}$ $\left[\frac{\text{m}}{\text{s}^2}\right]$	% relative uncertainty	$a^{measured}$ $\left[\frac{\text{m}}{\text{s}^2}\right]$	% relative uncertainty	$\frac{\lVert a^{meas} - a^{th} \rVert}{a^{th}}$
3.88 ± 0.01	0.119 ± <0.001	0.37 %	0.120 ± 0.0018	1.5 %	0.84 %
5.79 ± 0.01	0.176 ± <0.001	0.28 %	0.172 ± 0.0024	1.4 %	2.27 %
7.75 ± 0.01	0.234 ± <0.001	0.24 %	0.234 ± 0.0033	1.4 %	0.21 %

Table 2: Theoretical and Measured Relative Uncertainties

5 Conclusion

In this experiment we determined the acceleration along a frictionless horizontal ramp of a sliding object that has attached to it a rope from which ≈4g, ≈6g and ≈8g of mass suspended from it in two independent ways. The uncertainty in the theoretical value of acceleration for the ≈4g, ≈6g, and ≈8g masses were 0.37%, 0.28% and 0.24% respectively. The uncertainty in the measured value of acceleration for the ≈4g, ≈6g, and ≈8g masses were 1.5%, 1.4% and 1.4% respectively. The difference in acceleration is smaller than the total uncertainties, therefore the two values agree within their uncertainties.