Purpose of the lab experiment:

To measure the horizontal component of the Earth's Magnetic Field.

Theory

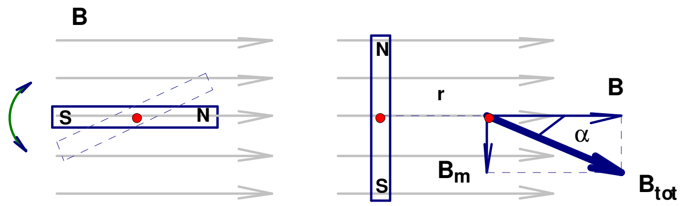

Fig. 1 Fig.2

A small bar magnet placed in the presence of external magnetic field, B, will allign itself along that field. At small angles, it will oscillate about its equilibrium position with a frequency equal to

$$f = \frac{1}{2\pi}\sqrt{\frac{\mu B}{I}}, \qquad \text{(Eq. 1)}$$

where I is the moment of inertia of the bar magnet about an axis through its center of mass, and μ is the magnetic dipole moment of the bar magnet. For a rectangularly shaped bar magnet,

$$I = \frac{ma^2}{12},$$

where a is the length of the magnet's side. The magnetic field produced by the bar magnet itself, at points sufficiently far from the magnet, is given by the following approximation:

$$B_m = \frac{\mu_o}{2\pi}\frac{\mu}{r^3}, \qquad \text{(Eq. 2)}$$

where μ_o is the permeability of free space, μ is the magnetic dipole moment, and r is the distance to the magnet. The total field, then, will be the vector sum of the Earth's magnetic field and the magnet's field, and in the case when both fields are perpendicular to each other, it will be:

$$B_{tot} = \sqrt{B_m^2 + B^2}.$$

The angle between the total field and the Earth's field can be measured by using a compass, whose needle always aligns itself along the total magnetic field at the point where it is placed. If the angle between the total field and the original direction, pointing along the horizontal component of the Earth's magnetic field is α, the relation between the Earth's magnetic field and the magnet's field is:

$$\frac{B_m}{B} = \tan\alpha. \qquad \text{(Eq. 3)}$$

If we combine Eqs. $(1)-(3)$, we can express the horizontal component of the Earth's magnetic field as a function of the frequency of oscillations, ν, the angle α, the distance r from the bar magnet to the compass, and the moment of inertia, I:

$$4\pi^2 f^2 I = \mu B = \mu \frac{B_m}{\tan\alpha} = \frac{\mu}{\tan\alpha}\frac{\mu_o}{2\pi}\frac{\mu}{r^3}$$

$$\mu = \sqrt{\frac{2\pi^2 f^2 r^3 I \tan(\alpha)}{\left(\frac{\mu_o}{4\pi}\right)}} \qquad\qquad B = \frac{\mu_o}{2\pi}\frac{\mu}{r^3}$$

Procedure

1. Calculate the moment of inertia
   - Measure the mass m - use the most precise scale (at least 3 digits)
   - Measure the side of the magnet a - use calipers
   - $I = \frac{ma^2}{12}$, $\frac{\triangle I}{I} = \frac{\triangle m}{m} + 2\frac{\triangle a}{a}$

	units	value	uncertainty	% uncertainty
mass, M	g			
side, a	cm			
inertia, I	kg			

2. Measure the frequency of the oscillations (7 times)
   - measure the time for 20 oscillations, t_{20}
   - calculate the period of the oscillations T
   - $f = \frac{1}{T}$, $\frac{\triangle f}{f} = \frac{\triangle T}{T} = \frac{\triangle t}{t}$

(sec)	1	2	3	4	5	6	7	aver.	σ	$\frac{\triangle t}{t}$
t										

	units	value	uncertainty	% uncertainty
T	s			
f	Hz			

3. Measure the angle at which the compass needle deflects
   - using the compass, determine the direction of Earth's field B
   - position the meter stick on top of the compass, $\perp$ to B
   - place the bar magnet on top of the meter stick, N pole points toward the compass
   - slide the magnet towards the compass until the needle deflects by 10°
   - write down the angle α
   - measure the distance between the magnet and the compass needle

	units	value	uncertainty	% uncertainty
r				
α				

Data Analysis:

$$\mu = \sqrt{\frac{8\pi^3 r^3 f^2 I \tan\alpha}{\mu_o}} \qquad\qquad B = \frac{\mu_o}{2\pi}\frac{\mu}{r^3}$$

I (kg.m^2)	% $\frac{\triangle I}{I}$	f(Hz)	%$\frac{\triangle f}{f}$	r (m)	% $\frac{\triangle r}{r}$	α	%$\frac{\triangle\alpha}{\alpha}$	μ (A.m^2)	B (T)

Calculation of error in the results:

$$\frac{\triangle\mu}{\mu} = \frac{1}{2}\left(3\frac{\triangle r}{r} + 2\frac{\triangle f}{f} + \frac{\triangle I}{I} + \frac{1}{\cos^2\alpha}\frac{\triangle\alpha}{\alpha}\right)$$

$$\frac{\triangle B}{B} = \frac{\triangle\mu}{\mu} + 3\frac{\triangle r}{r}$$

Final Results:

Give your answer with the error and the proper number of sig. figures.

$B =$ ____________ $\pm$ _____________ T